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Is the electromagnetic field in a medium a fluid or a wave?

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ABSTRACT

The electromagnetic field is found to travel inside a medium as a fluid that experiences a magnetic force on moving charges it encounters, and a drag (viscous) force. The field equations governing the dynamics of the fluid are found to be some altered Maxwell's equations. The energy conservation equation reveals that the energy flows is dissipationless, and lies in the plane containing the electric and magnetic fields only and no transverse flow. It is also conserved under the duality transformation. The magnetic field component along the velocity ($\vec{v}$) direction is conserved, while the electric field component is not. The electromagnetic field induces magnetization and polarization densities in the medium that are given by $\vec{M} = \varepsilon\,\vec{v} \times \vec{E}$ and $\vec{P} = \varepsilon\,\vec{v} \times \vec{B}$, respectively, where ε is the medium permittivity, and $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields. The electromagnetic field induces a local electric field that is equal and opposite to that one arised from a static cylindrical charge distribution.

1. Introduction

There is a intriguing analogy between the electrodynamics and hydrodynamics. The magnetic field is analogous to vorticity and the electric field is analogous to acceleration. In this analogy the Lorentz force equation is analogous to Euler equation [1]. Moreover, the compressibility condition is analogous to the Lorenz gauge condition. Owing to this analogy, Maxwell had treated the electromagnetic field as an incompressible fluid and the vector potential $\vec{A}$ as a velocity of some fluid particles not yet determined. The classical electrodynamics is thoroughly studied in Refs. [2,3].

We raise the question whether the fluidic formulation of electromagnetic field will lead to the same formulation as that done by Maxwell. While the energy of the fluid is carried by the fluid particles, the electromagnetic energy is carried away in a form of a wave traveling at the speed of light.

For this reason we do not expect the fluidic picture of the electromagnetic field to yield an energy equation that is analogous to Maxwell formulation. The type of the fluid particles must also be taken into consideration, where bosons are different from fermions. The electromagnetic field consists of an infinitely large number of photons. While a fluid propagates longitudinal sound waves through it, light waves have transverse nature.

Owing to duality principle, when light (photons) behave like particle it should be governed by fluid nature of the electromagnetic field, and a pure wave when governed by Maxwell's equations in vacuum. We therefore, trust that our current formulation will help to realize this complementarity nature of the electromagnetic field when quantum mechanical nature is presumed.

So what happens to light when it enters a medium? Does it flow as a fluid where it interacts with the medium, or it continues to behave as a wave? If it behaves as a particle then its dynamic should be described by equations that carries its

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evolution from the behavior of an electromagnetic field behaving as a fluid and not by a mere particle equation of motion. In this sense, we are exploring the mechanical aspect of the electromagnetic field. Recall that the electromagnetic field is found to posses energy and momentum, as do particles, and fluids. We don't compare here the hydrodynamics equations and make their connections with electrodynamics equations, but rather we treat the electromagnetic field as a fluid and explore its dynamics.

It is known that when a charged particle (q) encounters a stationary magnetic field, an interaction of the magnetic moment with the magnetic field arises. This occurs when we replace the momentum $\vec{p}$ by $\vec{p}+q\vec{A}$, where $\vec{A}$ is the magnetic vector, in the Hamiltonian of the charged particle. Here we have an electromagnetic fluid interacting with charge and current densities for which we replace $\vec{B}$ and $\vec{E}$ by $\vec{B}-\frac{\vec{v}}{v^2}\times\vec{E}$, and $\vec{E}+\vec{v}\times\vec{B}$, respectively. We do also expect here a similar interaction term in the energy equation to appear when an electromagnetic fluid encounters a charged medium. The magnetic moments of the medium charges interact with the dynamical magnetic field.

This paper is structured as follows: We introduce in Section 2 a fluidic nature of the electromagnetic field and the necessary equations. Here Maxwell's equations are expressed as total derivatives. In Section 3 we investigate the necessary conserved quantities pertaining to fluid formulation. We derive in Section 4 the wave equation, and derive the energy conservation equation of the electromagnetic fluid in Section 5. The paper ends with some concluding remarks in Section 6. Comparison between our present fluid formulation of the electromagnetic, as regards to the field, energy and momentum equations, and the Maxwellian analogues is shown in Appendix.

2. Fluidity of the electromagnetic field

We would like here to treat the electromagnetic field as a fluid and explore the dynamics of this fluid in terms of its electric and magnetic fields, and charge and current densities. To this aim, we should express Maxwell's equations as total derivatives of the electric and magnetic fields instead of partial derivatives. A normal fluid without viscosity is described by Euler equation. We will assume here the electromagnetic fluid is incompressible and irrotational. If we consider a fluidic nature of an electromagnetic field, one can write the total derivative as

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}\,, \tag{1}$$

where $\vec{v}$ is the fluid velocity. Now the Faraday's equation [2]

$$\frac{\partial\vec{B}}{\partial t}+\vec{\nabla}\times\vec{E}=0\,, \tag{2}$$

can be expressed as

$$\frac{d\vec{B}}{dt}=0\,, \tag{3}$$

where

$$\vec{\nabla}\times\vec{E}=(\vec{v}\cdot\vec{\nabla})\vec{B}. \tag{4}$$

Using the vector identity

$$\vec{\nabla}(\vec{A}\cdot\vec{C})=(\vec{A}\cdot\vec{\nabla})\vec{C}+(\vec{C}\cdot\vec{\nabla})\vec{A}+\vec{A}\times(\vec{\nabla}\times\vec{C})+\vec{C}\times(\vec{\nabla}\times\vec{A})\,, \tag{5}$$

the right hand-side of Eq. (4) yields

$$\vec{\nabla}\times\vec{E}=\vec{\nabla}(\vec{v}\cdot\vec{B})-\vec{v}\times(\vec{\nabla}\times\vec{B})\,, \tag{6}$$

where we have considered the velocity to be constant in space and time.

We now apply the Ampere's equation [2]

$$\vec{\nabla}\times\vec{B}=\mu\vec{J}+\frac{1}{v^2}\frac{\partial\vec{E}}{\partial t}\,, \tag{7}$$

in Eq. (4) using Eq. (2) to obtain

$$\frac{\partial}{\partial t}\left(\vec{B}-\frac{\vec{v}}{v^2}\times\vec{E}\right)+\vec{\nabla}(\vec{v}\cdot\vec{B})=\mu\,\vec{v}\times\vec{J}\,, \tag{8}$$

where v is speed of the electromagnetic field in the medium. Employing Eqs. (1) and (3), we can express the Ampere's equation, Eq. (7), as

$$\frac{\partial\vec{E}}{\partial t}+\vec{v}\cdot\vec{\nabla}\vec{E}=-\frac{\vec{J}}{\varepsilon},\quad \frac{d\vec{E}}{dt}=-\frac{\vec{J}}{\varepsilon}\,. \tag{9}$$