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Adiabatic invariant of electron motion in slowly varying magnetic field studied with the squeezing mechanism in entangled state representation

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ABSTRACT

Because the two coordinates of particle's guiding center of motion in uniform magnetic field do not commute, we think it is of necessity to study adiabatic invariant of electron motion in slowly varying magnetic field in the context of quantum mechanics. By constructing the entangled state representation which is the eigenvector of electron's coordinates and the squeezing mechanism of gyration radius, we directly reach the conclusion that the magnetic flux is adiabatic invariant in slowly varying magnetic field. We also compare this case with the adiabatic invariant of a pendulum whose string length is shortened very slowly.

1. Introduction

In quantum mechanics, an adiabatic change is one that occurs at a rate much slower than the difference in frequency between energy eigenstates. In this case, the energy states of the system do not make transitions, so that the quantum number is an adiabatic invariant. In early days of studying the theory of quantum mechanics some pioneers, e.g., Lorentz, Einstein, Bohr, et al. suggested that the quantities to be quantized (good quantum numbers) must be adiabatic invariants. Sommerfeld once summarized: "the quantum number of an arbitrary mechanical system is given by the adiabatic action variable." Sommerfeld once summarized: "the quantum number of an arbitrary mechanical system is given by the adiabatic action variable." As the first example of adiabatic invariant quantity, Lorentz considered a quantum pendulum whose string length l is shortened very slowly (adiabatically), the frequency of the pendulum changes, but the quantum number of the pendulum cannot change because at no point is there a high enough frequency to cause a transition between the states. Later Einstein pointed out that although the energy E and the frequency ν of the pendulum are both changed during the procedure, due to $\delta E/E = -\delta l/(2l)$, so their ratio $E/\nu\sim E\sqrt{l}$ is a constant (note a pendulum's period $2\pi\sqrt{l/g}$). This is analogous to Wien's observation that under slow motion of the wall the energy to frequency ratio of reflected waves is constant. Usually, the concept of adiabatic invariants introduced into quantum mechanics was recapitulated in the following way [1]:

In classical $x-p$ phase space, the possible path of a particle is confined in the curve

$$p(E,x) = \{2m(E-V)\}^{1/2}, \tag{1}$$

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where V is potential energy, this curve surrounds an area

$$\Phi(E) = p(E, x)\,dx. \tag{2}$$

Let the basic frequency of this motion be ν', then

$$\oint dt = \oint \frac{dx}{\nu'} = \oint \frac{\partial p}{\partial E}dx = \frac{d\Phi(E)}{dE}, \tag{3}$$

(note that $dE = \nu' dp$ is valid for any x), thus

$$\Phi(E) = \oint p dx, \tag{4}$$

so the frequency is

$$\nu' = \frac{dE}{d\Phi(E)}. \tag{5}$$

For a harmonic oscillator ν' is independent of E, thus

$$\Phi = \frac{E}{\nu'}, \tag{6}$$

is an adiabatic invariant quantity. Since the action variable for the harmonic oscillator is an integer n, Eq. (6) leads to the Bohr–Sommerfeld quantization rule:

$$\oint p dx = nh. \tag{7}$$

Eq. (7) is the foundation of the "Old Quantum Theory". Although this condition is not exact for the small quantum number n, and the whole theory is still semi-classical, it still gave good first thought to the correct way of quantization.

In a preceding paper [2] Fan and Chen have studied adiabatic invariant for quantized mesoscopic $L-C$ circuit which is composed of a capacity C and an inductance L connected to each other [3]. In this work we shall focus on finding adiabatic invariant for charged particle (with charge q) motion in time-varying magnetic field. This topic may have potential uses in various applications of plasma physics [4]. Considering a charged particle (with mass M and charge q) motion in the spatially uniform magnetic field in the $\hat{z}$ direction, $\vec{B} = B\hat{z}$, the cyclotron frequency is $\Omega = |q|B/M$, when B slowly increases within time, in the context of quantum theory we want to know what is the corresponding adiabatic invariant. Although this topic may also be studied in the context of classical electrodynamics, we still think it needs be tackled quantum mechanically, since from quantum mechanical view the two coordinates of guiding center of particle's trajectory do not commute, and the circular orbit concept of charged particles in uniform magnetic field is ambiguous.

Let us briefly review how to tackle this problem in the context of classical electrodynamics. From Maxwell equations a time-varying magnetic field engendered a space-varying electric field, $\nabla \times \vec{E} = -\partial\vec{B}/\partial t$, in cylindrical coordinates it is expressed as

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta) = -\frac{\partial B}{\partial t}, \tag{8}$$

where E_θ is induced along the motion path, noting B varies very slowly, $\partial B/\partial t$ is small, from Eq. (8) we have

$$E_\theta = -\frac{1}{2}r\frac{\partial B}{\partial t}, \tag{9}$$

or in vector form

$$\hat{E}_\theta = \frac{1}{2}\hat{r} \times \frac{\partial \hat{B}}{\partial t}, \tag{10}$$

the electric field will accelerates the particle, and the orbit is no longer a circle. However, since $\partial B/\partial t$ is small, E_θ is small too, and the orbit is nearly a closed circle. The increased transverse kinetic energy due to the induced electric field over one gyration period is

$$\delta\left(\frac{1}{2}Mv^2\right) = q\oint \hat{E}_\theta \cdot d\hat{r}. \tag{11}$$

Using Stokes's theorem, Eq. (11) becomes

$$q\iint \left(\nabla \times \hat{E}_\theta\right) \cdot d\hat{S} = -q\iint \frac{\partial \hat{B}}{\partial t} \cdot d\hat{S} = |q|\frac{\partial B}{\partial t}\pi R^2, \tag{12}$$