Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 199 (2017) 128–133

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Mohr Circle-based Graphical Vibration Analysis and Earthquake Response of Asymmetric Systems

Marco Faggella[a], Rosario Gigliotti[a], Carmen Morrone[a], Enrico Spacone[b]

[a]*Via Eudossiana 18, Rome 00184, Italy*
[b]*Viale Pindaro 42, Pescara 65129, Italy*

Abstract

The maximum seismic response of torsionally coupled plan asymmetric structures can be rationally visualized and computed through a Mohr Circle Response Spectrum Analysis (MCRSA). This is done combining the graphic modal properties of the torsional dynamic equations of motion with the structural earthquake demand in terms of a displacement spectrum as a function of the modal eigenvalues $S_D(\omega^2)$. A compact representation of the modal properties and of the response envelope is built and visualized in the Mohr plane. The maximum modal responses are then combined using a graphic adaptation of the SRSS and CCQ combination rules based on the elastic response spectrum. This Graphic Dynamic rule proves to be an effective response prediction tool, and is particularly suited to estimate the response of seismic base isolation systems.

Keywords: Graphic Dynamics; Asymmetric structures; Torsion; Mohr Circle; Earthquake Response Spectrum.

---

1. Introduction

In-plan irregularity and torsional coupling of structural systems are aspects of primary importance in seismic structural design and assessment. Several seismic codes predict design rules to incorporate torsional behavior that can be due to inherent eccentricity as well as accidental eccentricity, which covers different sources of asymmetric behavior. Torsional effects can often result in unbalanced demand on structural components leading to collapse or poor earthquake performance. This paper presents a graphic computation approach, the Mohr Circle Response Spectrum Analysis (MCRSA), for plan-asymmetric systems based on some new properties of the Mohr Circle. The method uses graphic dynamic properties and the Mohr Circle Modal Analysis (MCMA) to compute the modal system properties, [1, 2], and the Graphical Response Spectrum Analysis (GRSA), [3, 4], to determine the maximum modal response and combinations. Rigid floor diaphragm behavior is assumed to represent the rotational kinematics of the displacements and accelerations through the modal centers of rotation, referred to as the modal rotational pivots, [5, 6, 7].

1877-7058 © 2017 The Authors. Published by Elsevier Ltd.
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.194

The Mohr Circle approach provides useful insight into the dynamic behavior of torsionally coupled systems and a straightforward rule to predict the maximum dynamic torsional response of diaphragm systems. It can be particularly suited to predict the seismic response of base isolated systems, which have an inherent one-way only eccentricity and are characterized by rigid diaphragm behavior.

2. Mohr Circle- Modal Analysis (MCMA) and Response Spectrum Analysis (MCRSA)

Graphic Dynamic methods can be useful to highlight a number of geometric parameters governing the structural behavior and the free vibrations characteristics of torsionally coupled systems, [8 – 12], which can be identified on the Mass Circle of gyration and on the Ellipse of Elasticity, as it is shown in Figure 1a.

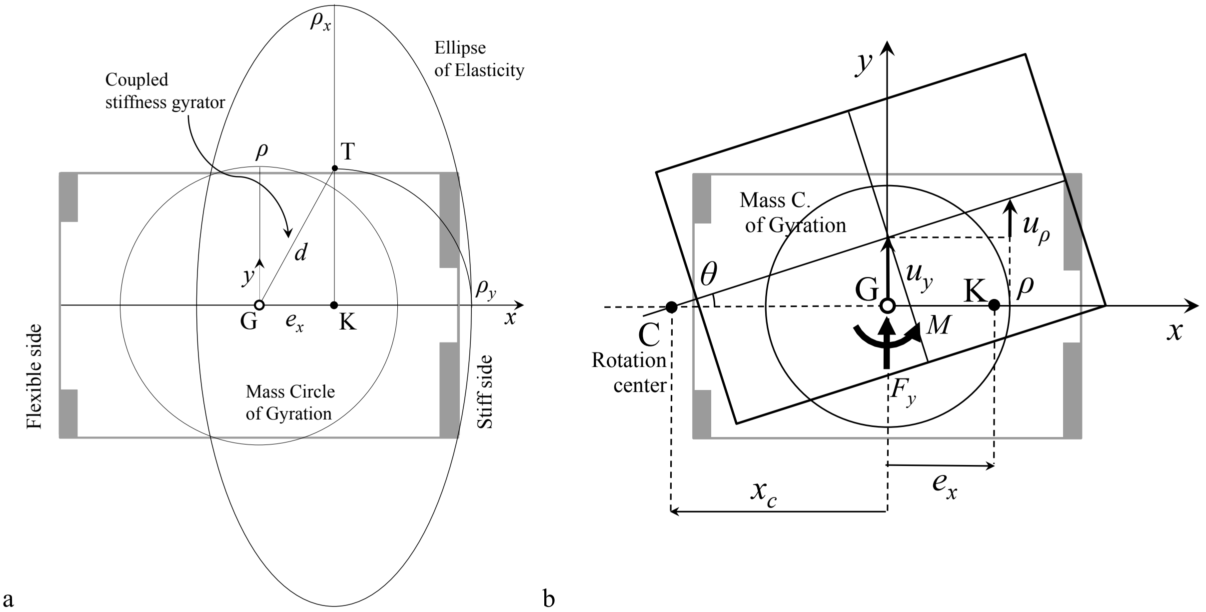

Fig. 1. (a) graphic dynamic properties of plan-eccentric torsional diaphragm systems; (b) rotational deformed shape about a center C, DOFs and equivalent translational DOFs

The first graphic indicator is the coupled stiffness gyrator *d*, which is the diagonal built on the eccentricity ex and on the semi-axis ρ_y of the Ellipse of Elasticity. The parameters relevant for the system of equations of dynamics are:

$$\rho_x=\sqrt{\frac{k_\theta}{k_x}},\quad \rho_y=\sqrt{\frac{k_\theta}{k_y}},\quad d=\sqrt{e_x^2+\rho_y^2},\quad \rho=\sqrt{\frac{I_p}{m}},\quad \Omega_s=\frac{\rho_y}{\rho},\quad q=\frac{d}{\rho},\quad \varepsilon_x=\frac{e_x}{\rho} \tag{1}$$

The parameters ρ_y and Ω_s are commonly used to measure the torsionally stiff vs. torsionally flexible character of the system, while the related parameters *d* and *q* account for both the in-plan distribution and magnitude of the stiffness of the lateral load resisting elements and for the additional torsional coupling induced by the eccentricity. It was noted that *d* figures directly in the rotational diagonal term of the stiffness matrix associated with the degrees of freedom u_y and θ, [2, 8], and represents a coupled stiffness gyrator. The dynamic system's characteristic polynomial equation can be solved in the graphic/nodal form, i.e. in terms of the position $x_{c,1}$ and $x_{c,2}$ of the modal centers of rotation C_1 and C_2 (modal pivots), which fully define the mode shapes. A dimensionless set of equations governing the dynamic torsional problem can be obtained assuming as degrees of freedom the displacement u_y at the center of mass G, and the torsional displacement $u_\rho=\rho\theta$, [13], associated with the dynamic stiffness matrix $\mathbf{k}_\rho$. The system