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# A finite element based method for estimating natural frequencies of locally damaged homogeneous beams

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## Abstract

Vibration-based damage detection from frequency changes requires the calculation of natural frequencies from assumed damage scenarios and conduct a comparison to the actual frequency of the structure. Analytical solutions in obtaining the natural frequency of homogeneous beams are currently limited to beams with uniform cross-sectional area. Changes in cross-sectional area might occur due to damage within the length of the beam. Finite element modeling and analysis is required in these instances, but may not be efficient in terms of computational effort. For the assumed damaged scenarios, there are unlimited number of possible damage combinations for which the natural frequency will be obtained. There is a need for an analytical alternative as a substitute to the finite element method to calculate these frequencies. This study presents an analytical method to estimate the natural frequencies of locally damaged homogeneous beams based on statistical data obtained from finite element modeling and analysis. The method proposes a multiplier function in terms of the extent of area reduction, length, and location of damage in order to estimate the damaged frequency. The function was derived using curve-fitting techniques of data obtained from finite element modeling and analysis of typical beams with assumed damage cases. Examples show that the method is a good alternative to finite element analysis in estimating the natural frequencies of locally damaged homogeneous beams. The method can be used for vibration-based structural health monitoring to predict the damage state of beams given the change in frequency without the computational burden of finite element modeling and analysis.

*Keywords:* natural frequency, finite element method, beam, cross-sectional area reduction, corrosion, vibration based damage detection

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## 1. Introduction

The ageing degradation of structures is the partial or total loss of their capacity to achieve the purpose for which they were constructed via a slow, progressive and irreversible process that occurs over a period of time [1]. Corrosion is the primary means by which metals deteriorate [2]. Some corrosion models have been developed such as the linear model [3,4,5] and the non-linear model [6] assume a corrosion rate that leads to a relationship between corrosion thickness and time. The effect of corrosion can be measured by the reduction of thickness in the material which led to quantitative measurement such as cross-sectional area loss which is adapted for this study [2]. Localized corrosion or other localized damage mechanisms can have more consequences than any other destructive processes individually and is considered to be more dangerous than uniform deterioration because it is more difficult to detect, predict and design against [7].

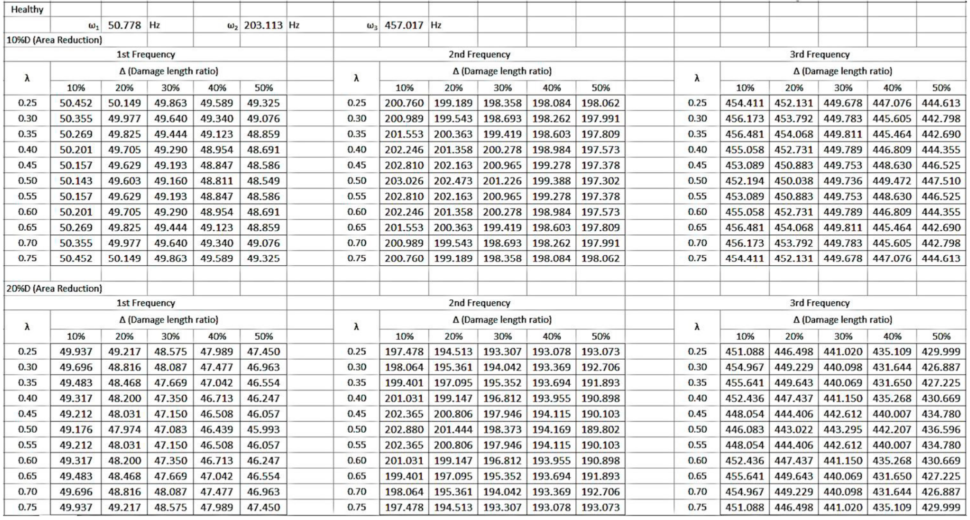
## 2. Vibration-based damage detection

The concept of vibration-based damage detection is that commonly measured modal parameters such as natural frequencies are functions of the physical properties of the structure such as the mass and stiffness. Therefore, changes in the physical properties such as the reduction in stiffness will cause detectable changes in the modal properties. These changes from the modal properties can be used as indicators of damage [8].

A typical procedure for conducting damage detection based on frequency changes on real world beam structures is composed of two phases - the forward problem, and the inverse problem. The forward problem consists of calculating frequency shifts from a known type of damage [8]. Table 1 shows the damaged natural frequencies produced by the forward problem governed by the three damage parameters damage length ratio *Δ*, damage location ratio *λ*, and the extent of damage $\alpha$. The theoretical calculation of damaged frequencies including the healthy frequency is traditionally done through finite element modeling and analysis.

The inverse problem consists of calculating the damage parameters from the frequency shifts [8]. It involves comparing the natural frequency of the actual real world beam to the pre-computed damaged frequencies of the forward problem. Using special procedures [9, 10], the frequency (i.e., damaged or healthy) that matches the frequency of the actual beam will be established as the most probable damage condition.

Table 1. Frequency table: Natural frequencies of a typical simple beam from different damage scenarios produced by the forward problem



| Healthy | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\omega_1$ | 50.778 | Hz | $\omega_2$ | 203.113 | Hz | | $\omega_3$ | 457.017 | Hz | | | | | | | |
| 10%D (Area Reduction) | | | | | | | | | | | | | | | | | |
| 1st Frequency | | | | | | 2nd Frequency | | | | | | 3rd Frequency | | | | | |
| λ | Δ (Damage length ratio) | | | | | λ | Δ (Damage length ratio) | | | | | λ | Δ (Damage length ratio) | | | | |
| | 10% | 20% | 30% | 40% | 50% | | 10% | 20% | 30% | 40% | 50% | | 10% | 20% | 30% | 40% | 50% |
| 0.25 | 50.452 | 50.149 | 49.863 | 49.589 | 49.325 | 0.25 | 200.760 | 199.189 | 198.358 | 198.084 | 198.062 | 0.25 | 454.411 | 452.131 | 449.678 | 447.076 | 444.613 |
| 0.30 | 50.355 | 49.977 | 49.640 | 49.340 | 49.076 | 0.30 | 200.989 | 199.543 | 198.693 | 198.262 | 197.991 | 0.30 | 456.173 | 453.792 | 449.783 | 445.605 | 442.798 |
| 0.35 | 50.269 | 49.825 | 49.444 | 49.123 | 48.859 | 0.35 | 201.553 | 200.363 | 199.419 | 198.603 | 197.809 | 0.35 | 456.481 | 454.068 | 449.811 | 445.464 | 442.690 |
| 0.40 | 50.201 | 49.705 | 49.290 | 48.954 | 48.691 | 0.40 | 202.246 | 201.358 | 200.278 | 198.984 | 197.573 | 0.40 | 455.058 | 452.731 | 449.789 | 446.809 | 444.355 |
| 0.45 | 50.157 | 49.629 | 49.193 | 48.847 | 48.586 | 0.45 | 202.810 | 202.163 | 200.965 | 199.278 | 197.378 | 0.45 | 453.089 | 450.883 | 449.753 | 448.630 | 446.525 |
| 0.50 | 50.143 | 49.603 | 49.160 | 48.811 | 48.549 | 0.50 | 203.026 | 202.473 | 201.226 | 199.388 | 197.302 | 0.50 | 452.194 | 450.038 | 449.736 | 449.472 | 447.510 |
| 0.55 | 50.157 | 49.629 | 49.193 | 48.847 | 48.586 | 0.55 | 202.810 | 202.163 | 200.965 | 199.278 | 197.378 | 0.55 | 453.089 | 450.883 | 449.753 | 448.630 | 446.525 |
| 0.60 | 50.201 | 49.705 | 49.290 | 48.954 | 48.691 | 0.60 | 202.246 | 201.358 | 200.278 | 198.984 | 197.573 | 0.60 | 455.058 | 452.731 | 449.789 | 446.809 | 444.355 |
| 0.65 | 50.269 | 49.825 | 49.444 | 49.123 | 48.859 | 0.65 | 201.553 | 200.363 | 199.419 | 198.603 | 197.809 | 0.65 | 456.481 | 454.068 | 449.811 | 445.464 | 442.690 |
| 0.70 | 50.355 | 49.977 | 49.640 | 49.340 | 49.076 | 0.70 | 200.989 | 199.543 | 198.693 | 198.262 | 197.991 | 0.70 | 456.173 | 453.792 | 449.783 | 445.605 | 442.798 |
| 0.75 | 50.452 | 50.149 | 49.863 | 49.589 | 49.325 | 0.75 | 200.760 | 199.189 | 198.358 | 198.084 | 198.062 | 0.75 | 454.411 | 452.131 | 449.678 | 447.076 | 444.613 |
| 20%D (Area Reduction) | | | | | | | | | | | | | | | | | |
| 1st Frequency | | | | | | 2nd Frequency | | | | | | 3rd Frequency | | | | | |
| λ | Δ (Damage length ratio) | | | | | λ | Δ (Damage length ratio) | | | | | λ | Δ (Damage length ratio) | | | | |
| | 10% | 20% | 30% | 40% | 50% | | 10% | 20% | 30% | 40% | 50% | | 10% | 20% | 30% | 40% | 50% |
| 0.25 | 49.937 | 49.217 | 48.575 | 47.989 | 47.450 | 0.25 | 197.478 | 194.513 | 193.307 | 193.078 | 193.073 | 0.25 | 451.088 | 446.498 | 441.020 | 435.109 | 429.999 |
| 0.30 | 49.696 | 48.816 | 48.087 | 47.477 | 46.963 | 0.30 | 198.064 | 195.361 | 194.042 | 193.369 | 192.706 | 0.30 | 454.967 | 449.229 | 440.098 | 431.644 | 426.887 |
| 0.35 | 49.483 | 48.468 | 47.669 | 47.042 | 46.554 | 0.35 | 199.401 | 197.095 | 195.352 | 193.694 | 191.893 | 0.35 | 455.641 | 449.643 | 440.069 | 431.650 | 427.225 |
| 0.40 | 49.317 | 48.200 | 47.350 | 46.713 | 46.247 | 0.40 | 201.031 | 199.147 | 196.812 | 193.955 | 190.898 | 0.40 | 452.436 | 447.437 | 441.150 | 435.268 | 430.669 |
| 0.45 | 49.212 | 48.031 | 47.150 | 46.508 | 46.057 | 0.45 | 202.365 | 200.806 | 197.946 | 194.115 | 190.103 | 0.45 | 448.054 | 444.406 | 442.612 | 440.007 | 434.780 |
| 0.50 | 49.176 | 47.974 | 47.083 | 46.439 | 45.993 | 0.50 | 202.880 | 201.444 | 198.373 | 194.169 | 189.802 | 0.50 | 446.083 | 443.022 | 443.295 | 442.207 | 436.596 |
| 0.55 | 49.212 | 48.031 | 47.150 | 46.508 | 46.057 | 0.55 | 202.365 | 200.806 | 197.946 | 194.115 | 190.103 | 0.55 | 448.054 | 444.406 | 442.612 | 440.007 | 434.780 |
| 0.60 | 49.317 | 48.200 | 47.350 | 46.713 | 46.247 | 0.60 | 201.031 | 199.147 | 196.812 | 193.955 | 190.898 | 0.60 | 452.436 | 447.437 | 441.150 | 435.268 | 430.669 |
| 0.65 | 49.483 | 48.468 | 47.669 | 47.042 | 46.554 | 0.65 | 199.401 | 197.095 | 195.352 | 193.694 | 191.893 | 0.65 | 455.641 | 449.643 | 440.069 | 431.650 | 427.225 |
| 0.70 | 49.696 | 48.816 | 48.087 | 47.477 | 46.963 | 0.70 | 198.064 | 195.361 | 194.042 | 193.369 | 192.706 | 0.70 | 454.967 | 449.229 | 440.098 | 431.644 | 426.887 |
| 0.75 | 49.937 | 49.217 | 48.575 | 47.989 | 47.450 | 0.75 | 197.478 | 194.513 | 193.307 | 193.078 | 193.073 | 0.75 | 451.088 | 446.498 | 441.020 | 435.109 | 429.999 |

## 3. Problem statement

The frequency-change sensitivity method relies on sensitivity matrices computed using finite element methods that requires substantial amounts of computer and user time [8, 11]. To obtain the required accuracy, most finite element