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Mathematical Modeling Applied to Sustainable Management of Marine Resources

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Abstract

In this work, we formulate and study a nonlinear mathematical model of fishery management to understand the dynamics of a fishery resource system in an aquatic environment that consists of two zones; one is free fishing zone and another is reserve zone where fishing is strictly prohibited. We have analyzed the model by finding the existence of equilibrium points: biological and bionomic, dynamical behavior of equilibrium points and also derived the conditions of stability and instability of the system. The behavior of this dynamical model of marine fishery management has been illustrated by the numerical simulations to establish the presented analytical results.

Keywords: mathematical model; natural resources; marine resources; fishery models; non-linear differential equations; numerical analysis

1. Introduction

In recent years, environmental issues including air and water pollution, climate change, overexploitation of marine ecosystems, exhaustion of fossil resources and conservation of biodiversity are receiving major attention from the public, stakeholders and scholars from the local to the planetary scales. It is now well recognized that human activities yield major ecological and environmental stresses with irreversible loss of species, destruction of habitat or climate catastrophes as the most dramatic examples of their effects. From this point of view, it is of great concern and environmental challenges today to preserve, conserve and manage the renewable natural resources in the marine and coastal areas for the sustainable development in Bangladesh. Sustainable development emphasizes the need to organize and control the dynamics and the complex interactions between man, production activities, and natural resources in order to promote their coexistence and their common evolution.

Efficient and sustainable management of natural resources and the control measure of such systems mainly depends on essential understanding the mechanisms of their evolution over time where mathematical modeling in terms of non-linear differential equations (ODEs) can play a significant role. As a result mathematical modeling is broadly used to discuss different types of real phenomena which lead to design better prediction, prevention, management and control strategies. For example, Biswas *et al.* [3] studied the potential impacts of Global Climate Change in Bangladesh.

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Mondal and Biswas [15] developed a mathematical model to describe the transmission of Nipah virus between bats and human. Neilan and Lenhart [17] showed the application optimal control strategy in disease modelling. Biswas and Haque [5] discussed necessary of nonlinear dynamical system to control the infectious disease. We also refer readers to [1, 4, 6, 8, 9, 16] and the references within for knowing the more applications of mathematical modeling.

During the last decades, many researchers presented their mathematical models as to optimizing fish production because of, mathematical modeling is the art of translating problems from an application area into traceable mathematical formulation with theoretical and numerical analysis. From of them, Leung and Wang [13] presented a mathematical model for commercial fishing, Clark et al. [7] studied the effects of irreversibility of capital investment upon optimal exploitation policies for renewable resources stocks, Kitabatake [11] developed a dynamic model for fishery resources with predator-prey relationship based on observational data for Lake Kasumigaura in Japan, Mesterton-Gibbons [14] also investigated an optimal policy to maximize the present value from the combined harvest of predator and prey, Zhang *et al.* [19] proposed and analyzed a model to study the optimal harvesting policy of a stage structured problem and derived necessary and sufficient condition for the coexistence and extinction of species. Recently, Song and Chen [18] discussed the optimal harvesting policy and stability for a two-species competitive system and derived conditions for the existence of a globally asymptotically stable positive equilibrium and a threshold of harvesting for the mature population, Dubey *et al.* [10] proposed a dynamic model for a single-species fishery which depends partially on a logistically growing resource.

From the literature discussed above and to the best of our views, we have proposed a mathematical model to optimize the production of fish with the help of system of nonlinear differential equation. We have discussed the equilibrium points, dynamical behavior of those points and obtained the conditions of stability and instability of the proposed model. Finally, we have discussed the numerical simulations with graphical representation of the proposed mathematical model.

2. Model formulation

We have considered a fishery resource consisting of two zones: a free fishing zone and a reserved area zone. We study a prey-predator system in a two patch environment, one accessible to both prey and predators and the other one being a refuge for the prey. Each patch is supposed to be homogenous. The prey refuge constitutes a reserve area of prey and no fishing is permitted in the reserved zone while the unreserved zone area is an open-access fishery zone. We suppose that the prey migrate between the two patches randomly. It is assumed that, in each zone growth of fish population follows logistic model. Keeping these in view, the model becomes-

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - \sigma_1 x + \sigma_2 y - qEx \tag{1}$$

$$\frac{dy}{dt} = sy\left(1 - \frac{Y}{L}\right) + \sigma_1 x - \sigma_2 y \tag{2}$$

Here, $x(t)$ be the biomass densities of the same population inside the unreserved area and $y(t)$ be the biomass densities of the same population inside the reserved area, respectively at a time t. Let E be the total effort applied for harvesting the fish population in the unreserved area and the value of qE is called fishing mortality. Again r and s are the intrinsic growth rate of fish subpopulation inside the unreserved and reserved areas respectively. Let the fish subpopulation of the unreserved area migrate into reserved area at a rate σ_1 and the fish subpopulation of the reserved area migrate into unreserved area at a rate σ_2. The crying capacities of fish species in unreserved and reserved area K and L respectively, q is the catch ability coefficient of fish species in the unreserved area. If there is no migration of fish population from reserved area to unreserved area ($i.e.\sigma_2 = 0$) and $r - \sigma_1 - qE < 0$, then $\frac{\partial x}{\partial t} < 0$. Similarly, if there is no migration of fish population from unreserved area to reserved area ($i.e.\sigma_1 = 0$) and $s - \sigma_2 < 0$, then $\frac{\partial y}{\partial t} < 0$.

3. Model analysis

The nonlinear system in equations (1)-(2) has qualitatively analyzed in this section so as to find the equilibrium points and the dynamical behavior of that points as well as test the stability at positivity analysis equilibrium points.