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ScienceDirect

Procedia Engineering 190 (2017) 170 – 177

Procedia Engineering

www.elsevier.com/locate/procedia

Structural and Physical Aspects of Construction Engineering

Microstructural Evaluation of Effective Elasticity Coefficients in Materials with Micro-voids

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Abstract

The paper deals with homogenization of linear elastic continuum involving empty voids. The concept of homogenization is meaningful provided that the size of voids is much smaller than the characteristic length in structural design. Assuming a uniform distribution of voids, one can utilize the concept of the representative volume element (RVE) which is a finite part of the macrostructure with involving one or more voids. In general, a numerical approach is necessary for solution of micro-structural boundary value problems in case of arbitrary shape and/or distribution of voids. Several approaches are discussed and mathematical models developed for numerical calculation of effective material coefficients for linear elastic continuum involving arbitrary empty voids. Appropriate micro-structural boundary value problems in the RVE are proposed for numerical analyses which are utilized for a posterior evaluation of effective material coefficients. Comparisons of results by various numerical approaches are discussed for the circular and elliptical pores.

Peer-review under responsibility of the organizing committee of SPACE 2016

Keywords: empty pores; homogenization; representative volume element; boundary densities compatibility; energy compatiblity; rigid body motion equivalence.

1. Introduction

The material media are not often homogeneous in engineering practice. Since the design is mostly the matter of numerical simulations, it is important to deal well with evaluation of effective material coefficients. If the size of

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Peer-review under responsibility of the organizing committee of SPACE 2016
doi:10.1016/j.proeng.2017.05.323

material defects is much smaller than the characteristic length in structural design, the concept of homogenization is meaningful and applicable. Although the subject of homogenization is classical (Voigt and Reuss mixture rules, Hashin and Shtrikman upper and lower bounds [2], self-consistency method [3], Mori-Tanaka method [4-6, 1]), there are still some open questions [7], e.g. the unproved Hill's and Mandel's conjecture statement. The idea of homogenization is applicable also to microscopically inhomogeneous materials as long as these materials are macroscopically or statistically homogeneous in considered macrostructure. Recently also multiscale models are utilized with different physical treatment on different length scales. In this paper, we shall distinguish between the macro-structure and micro-structure with using the linear elasticity as the physical base for description of phenomena on both dimensional scales. The difference consists in modelling micro-heterogeneities in micro-structure and characterization of macro-structure as statistically homogeneous linear elastic composite or defected materials based on the macroscopic or overall or equivalent elastic behavior.

In this paper, the attention is paid to question of selection of appropriate micro-structural boundary value problems (*ms-bvp*) for efficient evaluation of effective material coefficients in homogenized macrostructures.

2. Microstructural modelling and evaluation of effective material coefficients for homogenized macrostructure

In homogenization based on microstructural modelling the *ms-bvp* is solved in the representative volume element (RVE) within the elasticity theory with modelling the micro-structure. The RVE must obey the following requirements: (i) it is relatively small sample of the material, i.e. the constraints and loading on the surface of the macrostructure are uniform within the length l which is the linear size of the RVE ($l \ll L$ where L is the characteristic dimension of the macrostructure); (ii) it is sufficiently large as compared with the linear size of micro-inhomogeneity (a) in order the spatial wave-length ($\sim a$) of the stress and strain fluctuations about a mean value be small compared with l ($a \ll l$), and the effects of such fluctuations become insignificant within a few wave-lengths from the boundary of the RVE. Having solved micro-structural boundary value problems in RVE (the micro-constituents are assumed to be homogeneous elastic continua; the shape and distribution of micro-constituents or defects are abstracted from experiment), one can get volume averages of micro-fields over the RVE. The transition from micro- to macro-level (where the macro-structural problems could be solved in effective continuum which is macroscopically homogeneous) depends on finding the connections between suitably defined macro-variables and averages of micro-fields. There are two main questions: (i) how to define macro-variables and boundary data for the RVE in a physically meaningful way; (ii) whether and how the macro-variables (alone or in combinations) are related to the volume averages of their micro-counterparts.

Let us consider a macroscopically homogeneous body and denote by B the regular sub-region occupied by a RVE of the same microscopically inhomogeneous 2D body composed of the homogeneous skeleton $\Omega \subset B$ and empty voids $B - \Omega$ with $\partial B = \Gamma$, $\partial\Omega = \Gamma \cup \Gamma_0$, Γ_0 being the boundary of the RVE, skeleton and voids in the RVE, respectively. The volume average or the mean values of the field variables are

$$\left\langle f^{eff} \right\rangle_B = \frac{1}{|B|} \int_B f^{eff}(\mathbf{x}) d\Omega , \qquad \left\langle f^{ms} \right\rangle_\Omega = \frac{1}{|\Omega|} \int_\Omega f^{ms}(\mathbf{x}) d\Omega \quad . \tag{1}$$

where the former relationship is for the effective field variable $f^{eff}(\mathbf{x})$ defined in the whole B, while the latter is used for the field variable $f^{ms}(\mathbf{x})$ related to the elastic skeleton (matrix material) in micro-structural boundary value problems. The stiffness coefficients in the homogenized effective continuum are determined by the constitutive law

$$\begin{pmatrix} \sigma_{11}^{eff} \\ \sigma_{22}^{eff} \\ \sigma_{12}^{eff} \end{pmatrix} = \begin{pmatrix} c_{11}^{eff} & c_{12}^{eff} & c_{16}^{eff} \\ c_{21}^{eff} & c_{22}^{eff} & c_{26}^{eff} \\ c_{61}^{eff} & c_{62}^{eff} & c_{66}^{eff} \end{pmatrix} \begin{pmatrix} \varepsilon_{11}^{eff} \\ \varepsilon_{22}^{eff} \\ 2\varepsilon_{12}^{eff} \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \left\langle \sigma_{11}^{eff} \right\rangle_B \\ \left\langle \sigma_{22}^{eff} \right\rangle_B \\ \left\langle \sigma_{12}^{eff} \right\rangle_B \end{pmatrix} = \begin{pmatrix} c_{11}^{eff} & c_{12}^{eff} & c_{16}^{eff} \\ c_{21}^{eff} & c_{22}^{eff} & c_{26}^{eff} \\ c_{61}^{eff} & c_{62}^{eff} & c_{66}^{eff} \end{pmatrix} \begin{pmatrix} \left\langle \varepsilon_{11}^{eff} \right\rangle_B \\ \left\langle \varepsilon_{22}^{eff} \right\rangle_B \\ 2\left\langle \varepsilon_{12}^{eff} \right\rangle_B \end{pmatrix} . \tag{2}$$