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Transient Temperature Fields in Growing Bodies Subject to Discrete and Continuous Growth Regimes

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Abstract

The heat transfer problems for growing bodies is the is the subject of present research. Temperature distributions in growing bodies that appear during discrete and continuous growth are studied. The investigation is based on analytical and semianalytical solutions. Analytical solutions are of the form of spectral expansions. The applicability of the analytical solutions is limited to a narrow class for laws of evolution of growth boundaries. Semianalytical solutions have a wider range of applications. The calculation and analysis of temperature fields in the ball under the condition of central symmetry are provided. An analysis of the temperature behavior on the growth boundary shows that, depending on the accretion rate, the boundary can be considered as an isothermal boundary (for high values of the accretion rate) or a boundary with variable effective temperature determined in the process of solving the problem.

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Keywords: Growing bodies; continuous growth; discrete growth; heat transfer; analytical solution; eigenfunctions; semianalytical solution; Galerkin method.

1. Introduction

A lot of natural phenomena and artificial processes are accompanied by deformation, non-uniform heating and changing of the material composition of solids that flow simultaneously. Examples include: formation of sedimentary, biogenic, volcanic rocks; crystallization, in particular snowflakes formation; electroplating, physical and chemical vapor deposition, solidification of melt, welding. From the mechanical point of view such solids are classified as growing solids [1]–[3]. In order to determine the thermomechanical state of growing solids one have to take into account the history of growth or, better to say, the scenario of solid creation, even if locally the material is pure elastic (for example, belongs to a class of linear thermo-elastic materials). The mathematical description of this specific memory is manifested, in particular, by the incompatible distortion fields and corresponding residual stresses that can not be removed by any smooth deformation. In this regard the mathematical models of growing bodies have much in

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common with the models that appear in continual theory of defects. Such models require nonclassical methods for the statements of boundary problems, as well as for the construction of their solutions.

Unlike general theory of growing solids, linear thermomechanical problems for growing solids go back a long way. One of them is the heat conduction problem with a moving boundary that is studying for more than one hundred and fifty years. It was first formulated by Lam and Clapeyron in 1831. Notable research in this area belong to Joseph Stefan: he solved the problem while calculating how quickly an ice layer on water grows [4].

In classical Stefan problem the following conditions on a moving boundary (that separates the growing body and the environment) are stated: the temperature is continuous, whereas the heat flux has a discontinuity. The value of discontinuity is determined by the physical parameters of the formation of a new phase (for example, by the latent heat of crystallization). The generalized Stefan model assumes that the temperature at the interface is also discontinuous have been formulated in [5]. This model describes, in particular, the dynamics of deposition processes in a vacuum.

The subject of the present study is a growing linear thermoelastic deformable solid, whose growth is due to the continuous flow (evaporation) of the material onto the boundary, with account that the temperature of the boundary differs from the temperature of the deposed material. We suppose that at the moment of joining a material particle (that, from a physical point of view, should be seen as a set of atomic scale particles extensive enough to determine the thermodynamic variables) the temperature on the interface changes abruptly, carrying the elementary thermal shock similar to one in the the problem of V.I. Danilovskaya [6]. From this standpoint we study a model problem for a ball, which material composition varies over time due to continuous joining material to one of its faces. It is assumed that the growing surface being initially spherical remains spherical throughout the process of growth and moves progressive, generally, with a variable rate. Physically, this process corresponds to the idealized uniform deposition of material on the spherical substrate. It should be noted that such theoretical studies were carried out for the growing bodies of various canonical form, in particular for a ball [7,8], but statement of boundary conditions consensual with the thermomechanics of growth remained controversial. As a rule it was assumed that the temperature on the growing boundary coincides with the temperature of the incoming material (one can find the analogue with classical Stefan problem). However, a detailed study of non-stationary fields for joining of a large number of discrete thin layers showed that the model with prescribed temperature on the boundary gives adequate physical description of the processes only under the condition that the characteristic time of growing process is much greater than the characteristic time of relaxation of non-uniform thermal fields in the bulk of a growing solid [9]. In this regard it is appropriate to consider the growing solid with discontinuous conditions for both temperature and heat flux (analogue of the generalized Stefan problem), and the value of discontinuity relate to the characteristics of the growth process.

Formulation of the boundary condition on the growing border with the detailed description of the physical (and chemical) processes seems to be extremely complicate and beyond the scope of this paper. Present results should be regarded as a rough approximation, based on the following hypotheses.

1. Joining material is layered, i.e. for an infinitesimal period of time a layer of constant infinitesimal thickness joins to the body.
2. At the moment of joining the material layer changes its temperature on a finite value during the infinitesimal time interval, thereby causing an infinitesimal heat shock on a growing boundary.
3. The heat transfer with the environment does not occurs or the temperature of boundary is fixed.

Model based on these hypotheses can be considered as the idealization thin thermal barrier layer formed in a neighborhood the boundary of growth.

Some assessment of the applicability of such a model can be given by comparing of stress and strain rate fields with corresponded fields obtained in the framework of the model for discretely growing solids. To do this we consider the sequence of such problems increasing the number of layers and reducing their thickness. However, it should be understandable that step function and continuous function whose graphs are visually similar, will never be fully equivalent each other, and therefore discrete growth and continuous growth has qualitative difference (although perhaps not essential to the engineering application).