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# Contact problems at micro/nano scale with surface tension

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**Abstract**

When contact radius reduces to micro or nano scale, the influence of surface tension becomes important. In this paper, we summarize some of our works in this subject. Based on the solution of a point force acting on an elastic half space with surface tension, we consider the indentation on an elastic half space by rigid cylinder and sphere, respectively. The explicit relations of load depending on contact radius and indent depth are presented. Through a finite element approach accounting for surface energy, we verify the equivalence of the compression of an elastic sphere and the indentation of a half space, even with the presence of surface tension. Furthermore, we address the influence of surface tension on adhesive contact, and give the explicit expressions of contact radius and indent depth. These works are useful to accurately characterize the contact problems at micro and nano scale.



*Keywords:* Contact; surface tension; surface energy; adhesion.

## 1. Introduction

When the contact size decreases to microns or nanometers, many experiments demonstrated size-dependence of hardness.[1,2] For the indent depth ranging from 0.1 to 100 microns, the strain gradient theory successfully explained the size-dependent hardness.[2,3] However, when the indent depth is less than 100 nanometers, both atomistic simulations and experimental evaluations suggested that surface effects play a critical role.[4]

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Gurtin and Murdoch[5] incorporated surface energy into solid mechanics, and established the surface elasticity theory. This theory has been used to analyze the elastic field around nanosized inhomogeneities,[6] the effective elastic moduli of nanocomposites,[7] and the bending, vibration and buckling of nanowires.[8,9] Later, Huang and Sun[10] and Huang and Wang[11] formulated the surface elasticity at finite deformation and derived the size-dependent effective properties of hyperelastic solids.

The influence of surface effects on contact problems has long been concerned with. In 1978, Hajji[12] studied the axisymmetric indentation on an elastic half space with a pre-stressed membrane and derived the elastic solution for a concentrated force. Based on the surface elasticity theory, He and Lim[13] and Huang and Yu[14] formulated the three-dimensional and two-dimensional surface Green's function, respectively. Chen and Zhang[15] presented the corresponding surface Green's function for anti-plane shear deformation. Considering the influence of surface tension in deformed configuration, Wang and Feng[16] derived the elastic field induced by a concentrated force acting on a half plane. Recently, Gao *et al.*[17] addressed both surface tension and surface elasticity in contact problem, which revealed that the influence of surface tension is dominant over surface elasticity.

The classical Hertzian contact model does not account for attraction between two surfaces. JKR model[18] and DMT model[19] include the adhesive force within and outside contact region, respectively. The presence of surface tension induces a jump of normal stress across surface, which is different from the normal adhesive force. Recently, Xu *et al.*[20] and Hui *et al.*[21] addressed the impact of surface tension on adhesive contact.

In this paper, we summarize several works about the influence of surface tension on some fundamental contact problems, and append the explicit expressions between load and indent depth, for the convenience of their applications in experiments and engineering.

## 2. Contact between a rigid cylinder and an elastic half plane with surface tension

For two-dimensional plain strain problem, Wang and Feng[16] presented the elastic field for a point force $F$ acting normally on an isotropic elastic half plane with surface tension. The surface normal displacement is given by

$$w(x,0)=\frac{2F}{\pi E^*}\int_0^\infty \frac{1}{(s\xi+1)\xi}\left[\cos(x\xi)-\cos(r_0\xi)\right]\mathrm{d}\xi \tag{1}$$

where

$$s=\frac{2\tau^0}{E^*} \tag{2}$$

is an intrinsic material length indicating the length scale that surface tension can affect, $\tau^0$ is the surface tension, $E^*=2G/(1-\nu)$ is the composite elastic modulus of the considered material, $G$ and $\nu$ are the shear modulus and Poisson's ratio, respectively. For a compliant elastomer with $G$=1 MPa, $\nu$=0.4 and $\tau^0$=0.1 J/m$^2$, $s$ is about 60 nm.[22]

Consider the indentation of an elastic half plane by a rigid cylinder with a circular cross section of radius $R$. The axis of the cylinder is along the $y$-axis. A resultant line force $P$ is applied on the cylinder along the $z$-axis and results in a contact width $2a$ and an indent depth $\delta$, referring to a datum point at a distance $r_0$ to the origin. For more details, please refer to Long *et al.*[23].

Assume the pressure within the contact region as $p(t)$, then we can formulate the normal displacement on the surface, which satisfies the boundary condition within the contact region as

$$\frac{2}{\pi E^*}\int_{-a}^{a}\int_0^\infty \frac{\cos\left[(x-t)\xi\right]-\cos(r_0\xi)}{(s\xi+1)\xi}\,p(t)\mathrm{d}t\mathrm{d}\xi=\delta-\frac{x^2}{2R} \tag{3}$$

The sum of the pressure within the contact region equals to the external load $P$, that is