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Improving the global analysis of mechanical systems via parallel computation of basins of attraction

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Abstract

Numerical integrations represent a time-consuming element in the long-term dynamics analysis of mechanical systems. This limits the resolution of the computations and the dimension of the system to be investigated numerically. In fact, even pushing memory resources to their thresholds, only few tools can deal with higher-dimensional systems. This work illustrates, in a preliminary manner, the results that can be obtained reducing the aforementioned constraints thanks to the implementation of algorithms based on a parallel computing approach. In particular, by focusing on basins of attraction, four applications are discussed. i) The full domain of attraction for a four-dimensional (4D) system describing a linear oscillator coupled with a nonlinear absorber is calculated. ii) The variation of a safe basin with respect to the system dimension is then analyzed. It is highlighted how 4D and 3D analyses provide more confident results with respect to 2D analyses. iii) The parametric variation of a 2D system with a reduced step is performed by building a 3D representation which allows to highlight a smooth transition between the states. iv) A convergence study of a basin of attraction resolution is carried out. The integrity factor is used as a comparison measure.

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1. Introduction

In modern investigations of dynamical systems it is realized that it is no longer possible to determine only the attractors, other unstable orbits (saddles, homoclinic, heteroclinic, etc.), and to detect their local bifurcations[1]. In fact, even if a solution is stable, it may be not visible in practice because it has a small neighborhood of safe initial conditions[2]. Furthermore, the attractor may disappear suddenly, by means of a crisis[3]. These (and other, indeed) phenomena call for a global analysis of the system at hand, to be added to the "classical" local analyses - which have to be done in any case.

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The key tool for developing global analysis are the basins of attractions, which are the subsets of the phase space that share the same attractor, i.e. the ensemble of initial states whose orbits converge to the same attractor when the time goes to infinity.

The determination of basins of attraction by means of analytical techniques is possible only in few and paradigmatic cases. In general, they can be determined only by numerical techniques, and their sizes and shapes are thus *a priori* unpredictable: they can surround the attractor with a large and compact area, or be tight and stretched; can fill a portion of the state space by presenting an intermediate size with tongues towards certain directions, or be characterized by a riddled behaviour caused by the bubbling of attractors [4]; their boundaries can be smooth or fractal [5], etc.. Attempts to classify basins into general classes have been presented in literature [6] with the goal of permitting easier comparison.

The concept of basin of attraction has been later extended to the "safe basin", i.e. the subset of initial conditions sharing a common property, not necessarily the convergence toward an attractor [1,7].

Studying the properties (robustness, compactness, etc.) of basins of attraction (or, more generally, of safe basin) is the goal of the dynamical integrity theory, which has been recently developed starting from the observation that "classical" (Lyapunov) stability is not enough for practical purposes [7]. In fact, if the surrounding basin of attraction is not "large and compact" enough, even small perturbations - which are always present in everyday applications - can lead the system to a different attractor, resulting in a different, unexpected and often unwanted dynamical behaviour.

A key tool of dynamical integrity is the definition of a measure that provides magnitude of the safe basin, which faces with the problem of the intrinsically unsafe fractal basins (since they imply sensitivity to initial conditions, which is unwanted in common applications). Thus, in addition of the magnitude of the safe basin (so called GIM), other measures of the dynamical integrity have been introduced in the past to rule out the fractal parts [1,2,7,8].

The reduction of the safety and the erosion of the basins of attraction of a guyed tower model are evaluated in [9]. Dynamical integrity analysis of parallel-plate Micro-Electro-Mechanical Systems (MEMS) have also been performed in the recent past [10]. The erosion of the basin of attraction for electrostatic microactuators due to both the amplitude and the frequency of the actuation voltage has been evaluated by [11]. The device presents an high sensitivity to the initial conditions, and modification in the excitation leads to a reduction of the smoothness of the boundary of the basins.

The safety and dynamic integrity of a parametrically excited cylindrical shell is undertaken in [12] by analyzing the evolution of the various basins of attraction in the four-dimensional (4D) phase space. Projections of a 4D phase space, describing the oscillations and stability of the same mechanical model have been also proposed [13]: with the use of basins of attraction the authors highlight the instability phenomena that may arise under loading conditions such as a parametric excitation of flexural modes, and the escape phenomenon from the pre-buckling potential well.

By using 2D cross sections of the 5D basins of attraction, erosion profiles and integrity measures for the parametrically excited noncontacting atomic force microscopy problem are obtained in [14]. In [15] the sizes variation of basins of attraction is analysed in a periodically forced pendulum with oscillating support in the case of time-varying dissipation.

The importance of basins of attraction as a global analysis tool is proved by the countless applications in the engineering fields [16], although other examples such as in economics, are not uncommon [17]. Both continuous [18,19] and discrete [20] systems are tackled. Two coupled logistic maps presenting partially riddled basins in systems with chaotic saddle located between two attractors are shown in [21]. The dynamics of dices rolling is undertaken in [22] showing basins of attraction of different cube die faces.

Beside the massive use of basin of attraction, the state of the art is still essentially based on two dimensional cross sections also for high-dimension systems [23]. Globally, only sections of the real phase space are represented, which, incidentally, may lead to overlook rare attractors or chimera states, which can have an important role in the global system dynamics [24,25]. This is basically due to the limited computational resources available.

To advance in the direction of determining full basins of attraction of higher dimensional systems, various attempts have been done in the past [26,27].

The authors have contributed by developing an algorithm that exploits parallel computation for faster determination of higher order, fully dimensional, basins of attraction [28,29]. These works are continued here by applying the algorithm to some specific problems of interest in mechanical systems. The goal is *not* a thorough investigation of the basins of attraction, of their behaviour, and of the consequences for mechanical engineering. This is left for future work.