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Understanding arithmetic concepts: Does operation matter?

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ABSTRACT

Most research on children's arithmetic concepts is based on (a) additive concepts and (b) a single concept leading to possible limitations in current understanding about how children's knowledge of arithmetic concepts develops. In this study, both additive and multiplicative versions of six arithmetic concepts (identity, negation, commutativity, equivalence, inversion, and associativity) were investigated in Grades 5, 6, and 7. The multiplicative versions of the concepts were more weakly understood. No grade-related differences were found in conceptual knowledge, but older children were more accurate problem solvers. Individual differences were examined through cluster analyses. All children had a solid understanding of identity and negation. Some children had a strong understanding of all the concepts, both additive and multiplicative; some had a good understanding of equivalence or commutativity; and others had a weak understanding of commutativity, equivalence, inversion, and associativity. Associativity was the most difficult concept for all clusters. Grade did not predict cluster membership. Overall, these results demonstrate the breadth of individual variability in conceptual knowledge of arithmetic as well as the complexity in how children's understanding of arithmetic concepts develops.

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Introduction

What children know and understand about the four arithmetic operations is considered critical for developing successful mathematical skills (National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices, 2010). Children's knowledge of the arithmetic operations and their properties, including understanding the concepts and principles associated with addition, subtraction, multiplication, and division, is the foundation for learning basic arithmetic facts, effectively and flexibly using arithmetic problem-solving procedures, and developing later algebra and calculus skills (National Mathematics Advisory Panel, 2008). The vast majority of the research focuses on the operations of addition and subtraction (e.g., Crooks & Alibali, 2014; McNeil, 2014; Siegler & Araya, 2005). Additive concepts are the first to be learned by children and also are fundamental to learning the more complex operations of multiplication and division (Cowan & Renton, 1996; Kilpatrick, Swafford, & Findell, 2001). However, the greater complexity of multiplication and division has been the focus of less research despite the greater difficulties that children have with these two operations (Geary, 1994). Furthermore, there are few studies that directly compare children's understanding of additive and multiplicative concepts, making it difficult to determine exactly how additive and multiplicative concepts develop in relation to one another. For example, do children who understand an additive concept also understand the related multiplicative concept (e.g., Robinson, Ninowski, & Gray, 2006)? Finally, most studies are limited to one concept at a time, and only occasionally two or three concepts at a time, and which concepts are examined varies from one study to the next (e.g., Baroody, Lai, Li, & Baroody, 2009; Canobi, Reeve, & Pattison, 1998; Canobi, Reeve, & Pattison, 2002; Canobi, Reeve, & Pattison, 2003). This can lead to markedly different conclusions and theories on how arithmetic concepts develop because children's understanding of some concepts seems to develop early and in a fairly straightforward manner, whereas other concepts are still not strongly consolidated by adolescence or even adulthood (Baroody et al., 2009; Dubé, 2014).

The goal of the current study was to examine the overall understanding and development of both additive and multiplicative concepts and to do so using multiple concepts. This approach allows a richer and more complex portrait of how children's understanding of arithmetic operations and their properties develop and how this may vary across concepts and operations. In this study, we focused on six concepts that are considered integral to children's knowledge of arithmetic.

The concepts

The concept of identity involves understanding that subtracting zero from a number or dividing a number by one leaves the original number unchanged ($a - 0$ and $d \div 1$) (Baroody et al., 2009; Robinson, Dubé, & Beatch, 2017). The concept of negation involves understanding that subtracting a number from itself results in zero and that dividing a number by itself results in one ($a - a$ and $d \div d$). On word problems, a strong understanding of subtraction identity and negation exists by the time formal schooling begins (Baroody et al., 2009), and this has also been found with symbolic problems for Grade 3–5 children (Robinson et al., 2017). No research, to our knowledge, has examined the related division identity and negation concepts, but it is expected to be understood in Grades 3–5 (NCTM, 2000).

The concept of commutativity is the understanding that the order in which a pair of numbers is combined is irrelevant because any order yields the same answer ($a + b = b + a$ and $d \times e = e \times d$) and again is expected to be understood in Grades 3–5 (NCTM, 2000; NGACBP & CCSSO, 2010). Canobi (2005) found that nearly all 7- to 9-year-olds (roughly equivalent to Grades 2–4) understand commutativity on additive problems, yet Robinson et al. (2017) found that Grade 3–5 children had little understanding of commutativity in Grade 3 but that this increased to moderate understanding by Grade 5. The difference between the two studies may be accounted for by Canobi's participants being asked to judge whether a puppet using concrete objects had solved the problem correctly, whereas Robinson et al.'s participants were shown a symbolic problem (e.g., 4 + 7 = 7 + ?) and asked to solve it and then provide a verbal report of their solution strategy demonstrating their conceptual understanding of commutativity. Squire, Davies, and Bryant (2004) investigated the multiplicative version of commutativity in 9- and 10-year-olds (roughly equivalent to Grades 4 and 5) using word problems