Ecological Economics 133 (2017) 35–41

Contents lists available at ScienceDirect

Ecological Economics

journal homepage: www.elsevier.com/locate/ecolecon

Analysis

The Hotelling Rule for Entropy-constrained Economic Growth☆

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ARTICLE INFO

Article history:
Received 2 March 2016
Received in revised form 29 October 2016
Accepted 29 October 2016
Available online 1 December 2016

Keywords:
Thermodynamics
Entropy law
Hotelling Rule
Exhaustible resources

ABSTRACT

Lozada, Gabriel A. (1995), "xxx", Ecological Economics, ?: ???–???. The entropy change of the solar system between now and its final heat death is fixed. The time to the heat death is determined by the rate of entropy increase between now and then. If this rate of entropy increase is itself increased by economic activity, then economic activity is generating a negative externality. By internalizing this, a social planner treats the fixed amount of entropy change remaining until the heat death like the stock of an exhaustible resource. This leads to an analysis along the same lines as Hotelling's neoclassical economics of exhaustible resources, forming a partial synthesis between neoclassical economics and Nicholas Georgescu-Roegen's "ecological economics" work on the entropy law.

1. Introduction

Due to work such as Kåberger and Månsson (2001), Beard and Lozada (1999), Lozada (2004), Ayres (1998 p. 197), Floyd (2007), and Martyushev (2013), it has become clear that there is no elementary, intuitive interpretation of entropy. As Frank Lambert's article in the *Journal of Chemical Education* (2002) bluntly puts it, "Entropy is not disorder. Entropy is not a measure of disorder or chaos.". For example, when metallurgical slag and matte spontaneously separate, entropy becomes higher, disorder becomes lower, and economic usefulness becomes higher.

Nevertheless, it is undoubtedly true that all spontaneous processes increase entropy. It is also true that the entropic degradation of the Earth and the rest of the solar system will eventually result in the solar system's evolution to a "heat death" equilibrium, in which entropy has been maximized and therefore no further macroscopic physical processes are possible. If economic processes, by increasing the rate of entropic degradation, are bringing forward the date of that forbidding equilibrium state, then a problem of economic interest arises. Section 3 of this paper models that problem by formulating it within the standard neoclassical exhaustible-resource economics framework due to Hotelling (1931), though the definition of the limited resource is novel.

Glucina and Mayumi (2010 p. 22) warn that "delusions of grandeur" have characterized some writing about economics and entropy. To avoid that, we do not stop with Section 3's successes in showing that the idea of a "long-run entropic problem" is conceptually valid, but instead use Section 4 to ask a further question: is the long-run entropic problem empirically important? After all, economic problems can have constraints which are interesting in theory but which in a particular empirical setting are not binding, and thus are not important in that setting. Section 4 concludes that the long-run entropy problem's constraint is probably not binding. If further investigation supports that finding, then the long-run entropic problem, while potentially important, would not be actually important in practice.

Section 2 supplies background information to help interdisciplinary audiences understand Section 3, and Section 5 asserts that using mathematical models such as in Sections 3 and 4 is methodologically appropriate. Section 6 concludes.

The impetus for this paper came from the following passage written by Nicholas Georgescu-Roegen in one of the cornerstones of Ecological Economics:

> …let S denote the present stock of terrestrial low entropy and let r be some average annual amount of depletion. If we abstract (as we can safely do here) from the slow degradation of S, the *theoretical* maximum number of years until the complete exhaustion of that stock is S/r. This is also the number of years until the *industrial* phase in the evolution of mankind will forcibly come to its end. Give the fantastic disproportion between S and the flow of solar energy that reaches the globe annually, it is beyond question that, even with a very parsimonious use of S, the industrial phase of man's evolution will end long before the Sun will cease

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☆ I would like to especially thank an Editor of this Journal, Stefan Baumgärter, for very helpful and perceptive suggestions for improvement.

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http://dx.doi.org/10.1016/j.ecolecon.2016.10.022
0921-8009/

> to shine. … the fact remains that the higher the degree of economic development, the greater must be the annual depletion *r* and, hence, the shorter becomes the expected life of the human species. The upshot is clear. Every time we produce a Cadillac, we irrevocably destroy an amount of low entropy that could otherwise be used for producing a plow or a spade. In other words, every time we produce a Cadillac, we do it at the cost of decreasing the number of human lives in the future. (Georgescu-Roegen, 1980 pp. 57–58)

This excerpt has some flaws: its "beyond question" pessimism about solar energy actually is questionable. Also, its the notion of a "stock of … low entropy" is not quite right. However, merely by switching that notion to "a stock of a limited amount of entropy change", Section 3 obtains a physically-correct model of a long-run entropic problem, showing that Georgescu-Roegen's theoretical insight was mostly correct. On the other hand, Georgescu-Roegen also thought the long-run entropic problem was important in practice, which Section 4 casts doubt on. This paper makes future debate about whether Georgescu-Roegen was right or wrong on that point much easier, by showing that the question comes down to whether the shadow value of a particular constraint is close to zero.

Using the Söllner/Baumgärter classification system for papers incorporating thermodynamics into economics, this paper lies in Class 4c: "thermodynamic constraints on economic action: models incorporating entropy and entropy generation".[1]

2. Resolving Potential Interdisciplinary Misunderstandings

The first part of this preliminary section addresses misconceptions which may otherwise cause non-physicists to misunderstand the physics used in Section 3. The rest of this section clears up misconceptions which have caused natural scientists to think the mathematical framework used in Section 3 is wrong.

Georgescu-Roegen emphasizes the dialectical nature of the entropy law with turns of phrase such as "entropic indeterminateness". However, the entropy law *can* sometimes be used to obtain precise arithmomorphic results. It is used that way in Section 3, but since that is innovative, here is a non-innovative illustration. Consider a hypothetical chemical reaction $\mathrm{A}+\mathrm{B}\rightarrow 2\mathrm{C}$ where A, B, and C are perfect gases and where the reaction occurs at "standard" pressure (one atmosphere). Most chemical reactions do not go fully "to completion"; instead, some of the reactants remain in their initial form. The entropy law can be used to determine the precise equilibrium percent of completion. Lozada (1999 pp. 330–335) shows how. Briefly, if one supposes that the reaction starts with 1 mole of A and 1 mole of B, and if one lets n_A denote the number of moles of A which are left when the reaction reaches chemical equilibrium, then if the reaction occurs at constant temperature and pressure and the components freely mix, Lozada shows that

$$\Delta S = -R\left[2n_{\mathrm{A}}\ln\frac{n_{\mathrm{A}}}{2} + 2(1-n_{\mathrm{A}})\ln(1-n_{\mathrm{A}})\right] - (n_{\mathrm{A}}-1)\,(2S^{\circ}_{\mathrm{C}} - S^{\circ}_{\mathrm{A}} - S^{\circ}_{\mathrm{B}}) - \frac{1}{T}\left[-(n_{\mathrm{A}}-1)\,(2H^{\circ}_{\mathrm{C}} - H^{\circ}_{\mathrm{A}} - H^{\circ}_{\mathrm{B}})\right] \tag{1}$$

where ΔS is the change in entropy, R is the universal gas constant, and where S°_{A}, S°_{B}, S°_{C}, H°_{A}, H°_{B}, and H°_{C} are other constants characteristic of the substances A, B, and C. (The symbol S° denotes a substance's "standard entropy" and H° denotes its "standard enthalpy of formation"; if A, B, and C were real substances, one could look up their S° and H° in tables derived from laboratory experiments.) Lozada (op. cit., p. 334) continues (letting "J" stand for joules and °K for (degrees) Kelvin) (see also Beard and Lozada, 1999 p. 94):

> Equilibrium occurs in the state of maximum entropy, since from there, any deviation would decrease entropy and thus not be allowed by the entropy law. The state of maximum entropy is found by maximizing ΔS with respect to n_{A}. The value of R… [is approximately $8.314\mathrm{J}/(\mathrm{mol}\bullet{}^{\circ}\mathrm{K})$]. If in addition we assume for illustration that $T = 500^{\circ}\mathrm{K}$, $H^{\circ}_{\mathrm{A}} = 2500\mathrm{J/mol}$, $S^{\circ}_{\mathrm{A}} = 1\mathrm{J}/(\mathrm{mol}^{\circ}\mathrm{K})$, $H^{\circ}_{\mathrm{B}} = 2000\mathrm{J/mol}$, $S^{\circ}_{\mathrm{B}} = 2\mathrm{J}/(\mathrm{mol}^{\circ}\mathrm{K})$, $H^{\circ}_{\mathrm{C}} = 1000\mathrm{J/mol}$, and $S^{\circ}_{\mathrm{C}} = 4\mathrm{J}/(\mathrm{mol}^{\circ}\mathrm{K})$, then ΔS is maximized at $n_{\mathrm{A}} = 0.5229\ldots$. The reaction $A + B \rightarrow 2C$ will therefore go to $[(1-0.5529)*100=]$ 47.71 percent completion (cf. Gaskell, 1981 p. 230).

Section 3 does not try to characterize a thermodynamic equilibrium, as this example does, but it does take as given, *arithmomorphically*, that thermodynamic equilibrium is the state of maximum entropy.

Chemists and metallurgists almost always conduct calculations like those of the previous paragraph using Gibbs Free Energy instead of using entropy, but the entropy calculation is the more fundamental one—there is, tellingly, an "Entropy Law" but no "Gibbs Free Energy Law". The two calculations give exactly the same answer at constant temperature and pressure (Lozada op. cit. 346–7),[2] but as Lambert (2009) says, "the whole Gibbs relationship or function is about entropy change".

The above discussion shows that one *can* use entropy arithmomorphically, but does not address whether one *should* use entropy arithmomorphically. Section 5 addresses that.

Turning now to stumbling blocks in understanding economics: if an economist wishes to express the relationship between the amount of corn Q (in, say, liters) which is produced on a farm and the inputs water W (in liters) and fertilizer F (in kilograms) used to produce that corn, for almost a century a simple, popular choice has been the Cobb-Douglas functional form $Q = \gamma W^{\alpha} F^{\beta}$ where α, β, and γ are constants and the dimensions of γ are not discussed. All physical scientists are trained in dimensional analysis, from the perspective of which this expression for Q is incoherent: if γ has no dimensions then the left-hand side's "liters" is obviously not equal to the right-hand side's "liters to the α'' times "kilograms to the β". When economists write equations like $Q = \gamma W^{\alpha} F^{\beta}$, they know that what they really mean is

$$Q = \gamma W^{\alpha}\, F^{\beta} * 1\,\frac{\text{units of Q}}{(\text{units of W})^{\alpha}(\text{units of F})^{\beta}}$$

(assuming γ is dimensionless). It makes sense for economists to adopt the simplifying convention of never writing the last term because constants such as α and β are estimated from data and could be almost any real number (although a value between zero and one would generally have the most credibility). When for a particular farm one could obtain $\alpha = 0.2173$ and $\beta = 0.6894$, whereas for another farm α could be 0.8283 and β could be 0.1722, it is clear

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[1] The classification system is: (1) isomorphism of formal structure between thermodynamics and economics; (2) analogies and metaphors between thermodynamics and economics; (3) energy, entropy, and exergy theories of value; (4) thermodynamic constraints on economic action: (a) models incorporating mass and the conservation of mass, either for one particular material or for a number of materials; (b) models incorporating energy and the conservation of energy, sometimes in variants such as embodied energy; (c) models incorporating entropy and entropy generation; (d) models incorporating energy and entropy, sometimes in the form of exergy; and (e) models incorporating mass, energy, and entropy. See Baumgärtner (2004 pp. 112–6), who relies partially on Söllner (1997).

[2] The entropy calculation in Lozada (1999 p. 334) gives the same answer as the Gibbs Free Energy calculation not only Gaskell (1981 p. 230 line 3) but also in Gaskell (1995 p. 319 line 7) and in Gaskell (2008 p. 310 second line from the end).