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Existence and uniqueness of equilibrium in a distorted dynamic small open economy☆

Manjira Datta*

Department of Economics, Arizona State University, Tempe, AZ, USA

HIGHLIGHTS

- Unique recursive equilibrium exists for a distorted dynamic small open economy.
- New existence proof for recursive equilibrium.
- A small open economy mapped into known closed economy models with recursive equilibrium.

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ABSTRACT

We provide a set of sufficient conditions for existence and uniqueness of recursive equilibrium for a class of small open economies with distortion. The second welfare theorem is not available in this framework and usual approach for studying optimal neoclassical growth models is also not applicable.

1. Introduction

Trade theorists are interested in analyzing effects of liberalization on real income, economic growth or welfare in the presence of distortion, externalities and market frictions. For example, Ben-David and Loewy (2003) discuss open economy issues in the presence of knowledge spillover, Fernandez (2002) and Frankel (2003) consider effects of trade on environment and Kohn et al. (2016) analyze export dynamics under financial frictions. Willman (2004) provides a dynamic counterexample to gains from trade in the presence of strategic interaction. Equilibrium comparison in the presence of frictions is an important issue to understand. Yet, comparative static analysis is difficult due to multiple equilibrium and indeterminacy, see, for example, Nishimura and Shimomura (2002). One strand of the literature proceeds with assuming uniqueness of equilibrium e.g., Brown and Srinivasan (2007), and another strand of literature work with specific class of examples and computes the unique equilibrium, e.g., Datta (1997). This note provides insight on existence and uniqueness of recursive competitive equilibrium for a small nonoptimal open economy. In Section 2, we lay out our framework and note structural similarity with the closed nonoptimal economy studied by Datta et al. (2002) in which existence of a unique recursive competitive equilibrium is known. Section 3 describes the two-stage methodology and outlines main results.

2. The framework

We analyze an infinite horizon model of capital accumulation and trade: in the spirit of Brecher et al. (2005) and Datta (1992,

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* Fax: +1 480-965-0748
E-mail address: Manjira.Datta@asu.edu.

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1997) but modified to allow for policy-induced distortion and elastic labor supply. This framework can also be interpreted as a basic Uzawa model that allows for friction, externality or technological spillover. There is a continuum of identical household-firm agents. Each household owns an identical firm and operates under perfectly competitive conditions, both in the output and factors market. Datta et al. (2002) closed economy framework can be modified to analyze a small open economy with domestic market friction.

Two consumption goods denoted by c and m are tradeable. The small open economy only produces one good (c), using domestic factors of production: capital and labor. Distortion, externality or technological spillover are present. The second consumption good (m) must be imported and this is the only reason for trade in this framework. Capital is not traded and labor does not migrate. There is a labor-leisure trade-off: leisure (l) generates utility while labor (n) is essential for production and generating real income. Time is discrete, each period a representative household is endowed with capital (k) and a unit of time. Labor (n) is supplied elastically to the firm it owns. Output is produced with individual factors of production, (k, n), that the firm has direct control over but is also affected by per capita aggregates, (K, N), which it takes as given. The per capita aggregate variables generate technological externality, spillover and/or distortion. Each period firms rent capital and labor from the household, sell output (price normalized to one) in domestic competitive markets and return any profit and undepreciated capital back to the household. We assume following conditions on technology.[1]

Assumption 1. The production function, $f(k, n, K, N) : \mathbf{R}_+ \times \mathbf{R}_+ \times \mathbf{R}_+ \times \mathbf{R}_+ \rightarrow \mathbf{R}_+$, has private (individual) and social (per capita aggregate) factors in its domain such that
(a) f is constant returns to scale, increasing (strictly with each argument, for any non-zero input of the other), weakly concave (jointly, but strictly concave with each argument separately for any non-zero input of the other), and continuously differentiable in first two arguments with $f(0, n, K, N) = 0 = f(k, 0, K, N)$ for all $k, n, K, N > 0$;
(b) the marginal products of f in private capital and labor satisfy Inada-type conditions:

$$\lim_{k\to 0} f_1(k, n, K, N) \rightarrow \infty \text{ for all } n, K, N > 0,$$
$$\lim_{k\to \infty} f_1(k, n, K, N) \rightarrow 0 \text{ for all } n, K, N > 0,$$
$$\lim_{n\to 0} f_2(k, n, K, N) \rightarrow \infty \text{ for all } k, K, N > 0;$$

(c) there exists a maximal sustainable capital stock $k_{\max}$, such that $F(k, 1, K, 1) \leq k_{\max}$ *for all* $k, K \geq k_{\max}$.

Constant returns to scale in private returns ensure that the representative firm makes zero profit in each period. The representative household receives rental income, $r(K, N)k$, and wage income, $w(K, N)n$, which are distorted due to the presence of social externality and different from the social marginal returns a firms pay:

$$\hat{r}(K, N) = f_1(K, N, K, N), \qquad (1)$$
$$\hat{w}(K, N) = f_2(K, N, K, N).$$

We also allow for distortionary taxes to reflect market frictions that depend on the stock of capital, with the proceeds redistributed lump-sum to the households of the economy and define after-tax distorted factor prices as,

$$r = (1 - \pi_k(K))\hat{r}(K, N(K)),$$
$$w = (1 - \pi_n(K))\hat{w}(K, N(K)),$$

with lump-sum transfer to the household, $T(K)$. Distortions are allowed to have following characteristics:

Assumption 2. The vector of distortion functions, $\pi(K) = [\pi_k(K), \pi_n(K)] : \mathbf{K} \times \mathbf{K} \rightarrow [0, 1) \times [0, 1)$ are such that
(a) $r : \mathbf{K} \rightarrow \mathbf{R}_+$ is continuous, differentiable and decreasing in K with $\lim_{K\to 0} r(K) \rightarrow \infty$; and
(b) $w : \mathbf{K} \rightarrow \mathbf{R}_+$ is continuous, differentiable and (weakly) increasing in K.

Strong linear-logarithmic structure is imposed on the preference side to keep it simple but, more importantly, also to make sure the small open economy wants and is able to engage in trade every period despite the presence of domestic distortions. The household derives utility from consumption of domestically produced good, leisure and imported good, (c, l, m). Time is indexed by subscript $t \in \{0, 1, 2, \ldots\}$, and discount factor is constant, $\beta \in (0, 1)$. The objective is to maximize infinite horizon utility, subject to household budget and time constraints for $\alpha > 0$, $\gamma > 0$ and $\mu > 0$:

$$\sum_{t=0}^{\infty} \beta^t \{\alpha \log c_t + \gamma \log l_t + \mu \log m_t\}. \qquad (2)$$

Additive time separability allow us to solve the intertemporal optimization problem in two stages. One stage is a static optimization that focuses on intra-temporal choices and another on dynamic optimization with intertemporal choice as its focus, which is explained in detail in Section 3. Recursive competitive equilibrium is formally defined in appropriate function spaces that ties together individual utility maximization with market clearance and consistency between individual and aggregate variables. In what follows in the remainder of this section, we take advantage of linear-logarithmic preference structure, implications from one-period utility maximization and formally define recursive competitive equilibrium for a small open economy in the presence of distortion, externality or technological spillover.

If current investment and future capital (k') are known current domestic consumption, leisure and import decisions are made by maximizing current utility subject to budget constraint, $c + pm + k' = r(K, N)k + w(K, N)n + T(K)$, along with time constraint, $l + n = 1$. The constraints can be rewritten as, $c + pm + w(K, N)l = r(K, N)k + w(K, N) + T(K) - k'$, such that choice variables (along with respective prices) are on the left-hand side of the equation and all right-hand side terms are exogenous. Equating the ratio of marginal utilities of consumption of import and domestic good to the ratio of the respective prices we have,

$$\mu c = p\alpha m. \qquad (3)$$

Similarly, equating ratio of marginal utilities of current period domestic consumption to leisure to their respective price ratios we get,

$$\gamma c = w(K, N)\alpha l. \qquad (4)$$

The marginal rate of substitution between domestic consumption and leisure is a special case of Datta et al. (2002) but trade introduces a second marginal rate of substitution condition between consumption of domestic and imported goods. And, it is useful to note that the value of imported good demanded is proportional to the demand for domestic consumption good and that proportionality constant does not change over time. This allows us to define a composite consumption good, $z = c + pm = (\frac{\alpha+\mu}{\alpha})c$, and map the small open economy model we are studying in this note to the closed economy framework in Datta et al. (2002) and apply their methodology.

[1] These are not the weakest possible assumptions needed for our *existence* results. However, strong assumptions are required for *uniqueness* of equilibrium given the results of multiplicity and indeterminacy in the literature.