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Violations of monotonicity in evolutionary models with sample-based beliefs

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HIGHLIGHTS

- Not all evolutionary models exhibit the property of monotonicity.
- Violations of monotonicity arise when agents form beliefs by sampling.
- This is a consequence of the Central Limit Theorem.

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ABSTRACT

This paper considers a class of evolutionary game-theoretic models, namely those in which agents form beliefs about the behavior of others on the basis of random samples from the population. It shows that the dynamics of these models violate the property of monotonicity, which many authors have argued any well-specified evolutionary model should possess.

1. Introduction

This paper shows that an important class of game-theoretic models of evolution – those in which agents form beliefs about the behavior of others on the basis of samples taken from the population – violate a property – monotonicity – that many authors have argued any well-specified evolutionary model should possess.

In evolutionary models, each individual in a population exhibits one kind of behavior out of a set of possible behaviors. These individuals interact in a sequence of discrete periods or in continuous time, and some process or law of motion determining how the proportion of individuals exhibiting each kind of behavior changes over time is specified. A seminal contribution to this literature was Maynard Smith's (1982)[1] model of the evolution of behavior in (non-human) animal species. In this model, individuals are randomly matched into pairs every period, and a fitness function determines the number of offspring each individual produces, given its behavior and the behavior of the individual with whom it is matched. Offspring exhibit the same behavior as their parents. In applications of this model to human behavior, payoffs represent some measure of utility or profits, rather than reproductive success, and agents choose their actions with some degree of rationality, rather than being genetically programmed to adopt the same behavior as their parents.[2]

Many evolutionary models satisfy a property known as *monotonicity*. A variety of definitions of this notion have been introduced.[3] For 2×2 games, which are the focus of this paper, all

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1 See also Maynard Smith and Price (1973).

2 For book length surveys of this literature, see Fudenberg and Levine (1998), Samuelson (1997), Vega-Redondo (1996) and Weibull (1995). For surveys articles, see Mailath (1998a,b, 1992) and Van Damme (1994).

3 Weibull (1995) distinguishes among *payoff monotonicity* (p. 144), *payoff positivity* (p. 149) and *weak payoff positivity* (p. 151). Vega-Redondo (1996)

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of these notions imply that if, given the distribution of behaviors adopted by the population, a particular behavior yields a payoff that is higher than (respectively: lower than, equal to) the average for the population, then the proportion of the population exhibiting that behavior will increase (respectively: decrease, remain constant) over time. The property of monotonicity has strong intuitive appeal, and is satisfied in many evolutionary models, including Maynard Smith (1982). Many authors have in fact argued that monotonicity must be satisfied in any well-specified evolutionary model.

This paper explores violations of monotonicity in an important class of evolutionary models, namely those in which agents form beliefs about the behavior of others on the basis of samples of the past play of other individuals. We consider two models belonging to this class.

The first, which we refer to as *simple sample-based beliefs* (SSBB), provides a framework in which the nature of the violation of monotonicity we are studying is illustrated transparently. In that model, all members of a large population are randomly matched into pairs every period to play a coordination game; players base their beliefs about the behavior of their opponents on samples of the actions taken by members of the population who played the game in the immediately preceding period.[4]

Second, we consider a simple version of Young's (1993) model of *adaptive play*.[5] In that model, exactly two players from a large population are selected each period to play a coordination game; players' beliefs about opponents' behavior depend not only on the actions chosen by the pair that played the game in the immediately preceding period, but on samples drawn from all the pairs that played the game within some fixed number periods in the past.

In Section 2, we give a definition of monotonicity, and then state and prove a general theorem, showing that, in the SSBB model, this definition of monotonicity must be violated when the samples of previous period play observed by current period players are large. Section 3 explores the extent to which similar violations of monotonicity occur in adaptive play. We first demonstrate that, if we apply the definition given in Section 2, the non-monotonicity theorem for the SSBB does not carry over to adaptive play; in fact, we show that it is impossible to make general statements about whether or not adaptive play satisfies monotonicity as defined in Section 2. We then formulate a second definition of monotonicity – one that is equivalent to the definition given in Section 2 when applied to the SSBB, but not when applied to adaptive play – and show that by this definition, the violation of monotonicity that we demonstrated in the SSBB also necessarily arises in adaptive play.

2. Simple sample-based beliefs (SSBB)

2.1. The model

A large (even) number of players is randomly matched into pairs in each of an infinite sequence of discrete periods. In each period, each pair of players plays a symmetric game with strategy space $\{A, B\}$ and payoffs shown in Fig. 1. We assume that $a > c$ and $b < d$, so that we have a coordination game.

		Player 2: A	Player 2: B
Player 1	A	a, a	b, c
	B	c, b	d, d

Fig. 1. The payoff matrix.

Let p_t represent the proportion of players choosing A in any period t. Since any player has probability p_t of being matched with an A-chooser and probability $1 - p_t$ of being matched with a B-chooser,[6] the expected payoffs from choosing A and B, respectively, are

$$\begin{aligned} EU\,(A|p_t) &= p_t a + (1 - p_t)\, b \\ EU\,(B|p_t) &= p_t c + (1 - p_t)\, d. \end{aligned} \quad (1)$$

In each period, each player chooses an action that maximizes her current expected payoff.[7] Let $p^* \equiv \frac{d-b}{(a-c)+(d-b)}$ represent the value of p_t that equates the expected payoffs from playing A and B. Then if a player knew the true value of p_t, she would choose A if $p_t \geq p^*$ and she would choose B otherwise.[8]

But players are not able to observe the true proportion of A-choosers. Rather, they form beliefs about p_t on the basis of observations of other players' past behavior. In every period t, each player i observes a random sample[9] of s individuals from the population, and records the action of each individual in the sample in the previous period, $t - 1$. These samples are drawn independently across individuals. Let $\hat{p}_{it}$ represent the proportion of A-choosers in player i's sample in period t.

People then form beliefs adaptively: when player i chooses her action in period t, she assumes that the proportion of A-choosers that period will be equal to $\hat{p}_{it}$.[10] Hence, in period t, player i chooses A if the realization of $\hat{p}_{it}$ was at least p^* and chooses B otherwise. Given some true proportion p_t of A-choosers in period $t - 1$,[11] the probability that player i chooses A in period t can therefore be written as $\Pr_{it}\,(A|p_t) = \Pr\left(\hat{p}_{it} \geq p^*|p_t\right)$. Moreover, since the $\hat{p}_{it}$'s of all the players are independently and identically distributed,[12] we can drop the player subscript i and write the probability that any player chooses A in period t as $\Pr_t\,(A|p_t) = \Pr\left(\hat{p}_t \geq p^*|p_t\right)$, where $\hat{p}_t$ is the proportion of A-choosers in a random sample of size s from a population in which the true proportion of A-choosers is $\hat{p}_t$. Finally, it will be convenient to write this probability in the equivalent form of $\Pr_t\,(A|p_t) = \Pr\,(\Omega_t \geq sp^*|p_t)$, where $\Omega_t \equiv s\hat{p}_t$

distinguishes between *growth monotonicity* (p. 88) and *sign-preserving* dynamics (p. 88). Fudenberg and Levine (1998) distinguish between *payoff monotonicity* (p. 74) and *aggregate monotonicity* (p. 76). For 2 × 2 games, all of these definitions are equivalent to the definition given in Section 2.2 below.

4 Ball (2016) studies a model of the joint evolution of social norms and economic prosperity in which beliefs are formed in this way; Durieu and Salal (2003) develop such a model with a spatial dimension, in which each player samples the previous period play of neighbors located within a certain distance.

5 See also Young (1996, 1998). Hurkens (1995) also studies a model in this class. Fudenberg and Levine (1998, pp. 114–117) give an overview of these models, which they call games with "partial sampling".

6 We assume that the population is large enough that each player can ignore the effect of her own action on the population distribution.

7 As in Young (1993) and much of the related literature, players do not consider how their current actions might influence the beliefs and actions of others in the future.

8 We are making the arbitrary assumption that if a player is indifferent between A and B she chooses A. All of the following analysis would go through if players indifferent between A and B were assumed to choose B or to randomize between A and B with any weights.

9 We assume that these samples are drawn with replacement, so the draws within any sample are independent, and the number of A-choosers in any individual's sample follows a binomial distribution. Of course, if we assumed that sampling was done without replacement [as in Young, 1993], all of the analysis would go through provided the population was large relative to the sample size.

10 This assumption of adaptive expectations is central to much of the literature on evolutionary game theory. See Young (1996, pp. 107–112) and Kandori et al. (1993, pp. 30–32), for discussion.

11 We use the subscript t (rather than $t - 1$) for the proportion of players who chose action A in period $t - 1$ to emphasize that this is the proportion of A-choosers in the population from which players draw their samples in period t.

12 Because the players observe independent random samples of identical size from a common population.