Economics Letters 151 (2017) 35–38

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Solving and simulating unbalanced growth models using linearization about the current state☆

Kerk L. Phillips

Brigham Young University, Department of Economics, 166 FOB, Provo, UT 84602

HIGHLIGHTS

- We detail a method of solving and simulating DSGE models which is more accurate than the traditional method of linearizing about the steady state.
- Our method takes longer, but is able to sole and simulate unbalanced growth models which cannot be solved using any other current method.
- While the execution time is longer it is not prohibitively so, and the results are much more accurate, at least when measured by Euler errors.

ARTICLE INFO

Article history:
Received 6 September 2016
Received in revised form 30 October 2016
Accepted 25 November 2016
Available online 6 December 2016

JEL classification:
C63
C68
E37

Keywords:
Dynamic stochastic general equilibrium
Linearization methods
Numerical simulation
Computational techniques
Simulation modeling
Unbalanced growth

ABSTRACT

This paper presents an adjustment to commonly used linear approximation methods for dynamic stochastic general equilibrium (DSGE) models. Policy functions approximated around the steady state will be inaccurate away from the steady state. In some cases the model may not have a well-defined steady state, or the nature of the steady state may be at odds with its off-steady-state dynamics. We show how to simulate a DSGE model with no well-defined steady state by approximating about the current state. Our method minimizes the error associated with a finite-order Taylor-series expansion of the model's characterizing equations. This method is easily implemented and has the advantage of mimicking highly non-linear behavior. It also requires choosing N out of $2N$ possible roots from a matrix quadratic equations and the choice of this root not obvious away from the steady state. However, simulations show that using the same criteria as when linearizing about the steady state yield reasonable, well-fitting results.

1. Introduction

Dynamic stochastic general equilibrium (DSGE) models are an important class of macroeconomic modeling that have been in use now for several decades. They are increasingly used in policy contexts to simulate the effects of policy changes on the macroeconomy.[1]

Usually these models are too complex to find closed-form solutions for dynamic policy functions that map the current state of the economy into the values for next period's endogenous state variables. Instead, these models must be solved and simulated using some approximation method. The most widely used techniques include the linearization methods in Uhlig (1999) and Christiano (2002) who employ a method of undetermined coefficients to solve the state-space representation outlined in Blanchard and Kahn. (1980). Higher-order polynomial approximations developed by Judd (1992), Guu and Judd (2001), Collard and Juillard (2001) and Schmitt-Grohe and Uribe (2004) are increasingly widely used. This is the approach taken with the poplar DSGE software package Dynare, for example.[2]

This paper presents an easy adjustment to linear and higher-order approximation methods. Since approximation is almost always done about the model's steady state, the linear policy functions can be inaccurate if the simulation is often away from the steady state. In some cases, this leads to only small errors. In other cases, however, the model may not have a well-defined steady state, or the nature of the steady state may be at odds with its off-steady-state dynamics. Approximating about the steady state

☆ This research benefited from the computing resources of the BYU Macroeconomics and Computational Laboratory. Thanks to Chase Coleman, Mary Li, Yulong Li and Jake Orchard for their excellent research assistance and to Richard W. Evans for insightful comments and support.

E-mail address: kerk_phillips@byu.edu.

[1] See for example Smets and Wouters (2007) and Christiano et al. (2005).

[2] See Adjemian et al. (2014) for details.

http://dx.doi.org/10.1016/j.econlet.2016.11.035
0165-1765/

requires the existence of a steady state. If one does not exist, the model's variables must be redefined so that they are stationary. We refer to this method as "steady state linearization" (SSL).

Some models of interest to researchers cannot be easily transformed in this manner, however. These include multi-sector models with unbalanced growth and models where the parameters are time-varying. We show how to simulate an unbalanced DSGE model by approximating about the current state, rather than the steady state. This method is easily implemented and has the advantage of accurately mimicking non-linear behavior. The ability to accurately solve and simulate models that have no steady state makes it possible to examine the behavior of a richer class of models that may be better able to mimic the real economy.

We proceed similarly to Uhlig (1999) and Christiano (2002) who use SSL. They hypothesize linear policy functions and use the method of undetermined coefficients to solve for the coefficients of the policy function. We propose an alternative strategy, which we call "current state linearization" (CSL), where one approximates the policy function for each period of the simulation about the current state of the economy. This is computationally more intensive as it requires linearization each period, rather than only once. However, this method has the advantage of being much more accurate when the economy is far from the steady state and being feasible when no steady state exists. CSL can easily replicate the behavior of highly nonlinear policy functions. This is because the Taylor-series approximation is highly accurate in the neighborhood of the point about which a function is approximated. Since we are always approximating about the current state, our linear policy function will be very close to the true policy function for that state.

One major hurdle in polynomial approximations is choosing the appropriate roots from a matrix quadratic equation. In the steady state this is relatively simple since the system is generally stable about the steady state. Most macroeconomic models yield a unique set of roots that imply stability in that neighborhood. The same is not true away from the steady state. However, we show that for our simple unbalanced growth model, the criterion used for steady state stability works well, generating very small Euler errors during simulation. There is no guarantee that the roots with the smallest modulus correspond to the policy functions associated with the saddle path. Fortunately, with the simple illustrative model in this paper that criterion performs well.

While we illustrate our method using linear approximations, the concepts and method apply to higher-order polynomial approximations as well.

2. Derivation of an approximation about the current state

Consider a set of nonlinear expectational functions, in our case from a dynamic general equilibrium model. The state variables are grouped into two categories: exogenous state variables are grouped into the $S \times 1$ column vector, Z_t, while endogenous ones are placed in the $K \times 1$ column vector, X_t. There are K equations and they can be represented as in Eq. (2.1).

$$E_t\{\Gamma(X_{t+2}, X_{t+1}, X_t, Z_{t+1}, Z_t)\} = 0 \tag{2.1}$$

This system of equations can be approximated by taking a first-order Taylor series expansion about an arbitrary point in the state space. We choose the current value for the state variables, $\theta_t = \{X_t, Z_t\}$. This transformation is given in Eq. (2.2).

$$E_t\{T_t + F_t\tilde{X}_{t+2} + G_t\tilde{X}_{t+1} + H_t\tilde{X}_t + L_t\tilde{Z}_{t+1} + M_t\tilde{Z}_t\} = 0 \tag{2.2}$$

In the above equation, F_t, G_t and H_t are $K \times K$ matrices, L_t and M_t are $K \times S$ matrices, and T_t is an $K \times 1$ vector. All these will depend on which point is chosen for the linearization. Tildes denote absolute deviations from θ_t values. Note that if we choose to linearize about the steady state, $\bar{\theta} = \{\bar{X}, \bar{Z}\}$ the value of T_t is zero. While this is true of the steady state, it will not be true generally.

The law of motion for the exogenous state variables is assumed to be a first-order vector autoregression of the form in Eq. (2.3).

$$Z_{t+1} = (I - N)\bar{Z} + NZ_t + E_{t+1} \tag{2.3}$$

Since we are allowing for linearization around any value of Z, we proceed to transform (2.3) into (2.4).

$$E_t\{\tilde{Z}_{t+1}\} = Z_t - \bar{Z} \tag{2.4}$$

As with standard linearization techniques, our goal is to find a linear approximation to the policy function, (2.5).

$$\tilde{X}_{t+1} = U_t + P_t\tilde{X}_t + Q_t\tilde{Z}_t \tag{2.5}$$

where U_t is an $K \times 1$ column vector, P_t is an $K \times K$ matrix and Q_t is $K \times S$.

The major differences between (2.5) and the standard linear policy function are: First, the inclusion of the constant term, U_t, which makes it possible for the endogenous state variables to drift away from the current state. And second, the time-varying nature of the parameters P_t, Q_t and U_t. Iterative substitution of (2.4) and (2.5) into (2.2) yields the following three conditions which define P_t, Q_t & U_t.

$$F_tP_t^2 + G_tP_t + H_t = 0 \tag{2.6}$$

$$(F_tQ_t + L_t)N + (F_tP_t + G_t)Q_t + M_t = 0 \tag{2.7}$$

$$T_t + [F_tU_t + F_tP_tU_t) + G_tU_t + (F_tQ_t + L_t)(N - I)(Z_t - \bar{Z}) = 0 \tag{2.8}$$

The CSL method allows us to solve for each period's coefficients in isolation, without having to refer to next period's actual values. It has the advantage of not needing to solve for a benchmark time path via some other method. Indeed, it is not even necessary to solve for the steady state. Instead, we generate the time path as we solve and simulate each period. A disadvantage is that we must recalculate the unique values of P_t, Q_t & U_t each period in each simulation.

Choosing the roots from Eq. (2.6) is often straightforward when linearizing about the steady state as there is usually a unique set of roots that implies stability of the system. This need not be the case for CSL, however. Stability of the endogenous state variables need not hold away from the steady state. The true roots may imply a path that would be unstable were one to use them for the entire simulation. However, since they are used only for one period the instability is purely temporary and part of the convergence process. We show in our unbalanced growth example below, that choosing he roots with the smallest modulus works very well for CSL for our particular model. Unfortunately, this need not generally be the case.

3. Applying this method to an unbalanced growth model

In this section we consider a model with a labor leisure decision and technical progress. The household's problem is shown below. We adopt preferences as laid out in Jaimovich and Rebelo (2009).

$$V(k_t, x_{t-1}, z_t) = \max_{k_{t+1}, h_t} \frac{(c_t - \psi h_t^{\theta} x_t)^{1-\sigma}}{1-\sigma} + \beta E_t\left[V(k_{t+1}, x_t, z_{t+1})\right]$$

$$x_t = c_t^{\gamma} x_{t-1}^{1-\gamma} \tag{3.1}$$

$$c_t = w_t h_t + (1 + r_t - \delta)k_t - k_{t+1} \tag{3.2}$$