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Time-inconsistent preferences, investment and asset pricing[☆]

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HIGHLIGHTS

- We extend the production-based asset pricing model by incorporating time-inconsistent preferences.
- Time-inconsistent preferences induce under-investment, over-consumption and higher risk-free rate.
- The naïve agents consume more and invest less than the sophisticated agents.
- The interest rate in the economy with naïve agents is higher than that in the economy with sophisticated agents.

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ABSTRACT

In this paper, we present a production-based asset pricing model in which agents have time-inconsistent preferences. We find that the time-inconsistent preferences lead to under-investment, over-consumption, and higher interest rate. These variables are distorted more in the economy with naive agent than the economy with sophisticated agent. In particular, the sophisticated agent invests more and consumes less than the naive agent, but invests less and consumes more than the time-consistent agent. The interest rate in the sophisticated agent economy is lower than that in the naive agent economy, but higher than that in the time-consistent agent economy.

1. Introduction

The existing production-based asset pricing models assume that agents have constant time preferences. However, the evidence suggests that agents' time preferences vary over time (e.g., Thaler, 1981; Ainslie, 1992; and Loewenstein and Prelec, 1992). Time-inconsistent preferences imply that agents act relatively patiently when two payoffs are far away in time, but more impatiently when they are brought forward in time. Laibson (1997) models such time-varying impatience with a quasi-hyperbolic discount function, in which the discount rate declines with the horizon.[1] Harris and Laibson (2013) capture this effect with a continuous-time model. Grenadier and Wang (2007) extend the real op-

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[1] Laibson (1997) uses a discrete-time discount function to model quasi-hyperbolic preferences, in which time is divided into two periods: the present period and all future periods. Utility in the current period is discounted exponentially with a discount rate ρ, while utility in all future periods is first discounted exponentially with discount rate ρ and then discounted by an additional factor $\alpha \in (0, 1]$.

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tion approach to model the investment-timing decisions by entrepreneurs who have time-inconsistent preferences. Unlike these papers, we present a production-based asset pricing model in which capital adjustment costs are continuous-time quadratic (e.g., Abel and Eberly, 1994; Eberly and Wang, 2009; and Pindyck and Wang, 2013) and agents have time-inconsistent preferences, and then examine agents' consumption, asset prices and investment.

We analyze two economies with either sophisticated agent or naive agent. The sophisticated agent correctly realizes that his future selves act according to their own preferences, while the naive agent has wrong belief that the future selves act in the interest of the current self. We have three findings. First, naive agent consumes more than sophisticated agent, and both of them consume more than time-consistent agent. Second, naive agent invests less than sophisticated agent, and both of them invest less than time-consistent agent. Finally, the time-inconsistent preferences lead to a higher interest rate. Therefore, by considering time-inconsistent preferences, this paper enriches the existing asset pricing models.

2. Model setup

We consider a continuous-time, production economy in which agents have quasi-hyperbolic preferences, capturing the fact that the discount rate declines over time. Following Grenadier and Wang (2007) and Harris and Laibson (2013), we assume that each period has a random lifespan, which is modeled as a Poisson process with intensity of λ. Solving such a problem with time-inconsistent preferences can be thought as the outcome of an intrapersonal game, in which the agent is represented by different selves at future periods. Each self makes consumption–investment decisions during his lifetime but also concerns with the utility received by his future selves.

Let $D_n(t,s)$ denote self n's discount function. At time $t \in [t_n, t_{n+1})$, self n evaluates the utility received at future time s as $D_n(t,s)$ given by

$$D_n(t,s) = \begin{cases} e^{-\rho(s-t)} & \text{if } s \in [t_n,\ t_{n+1}) \\ \alpha e^{-\rho(s-t)} & \text{if } s \in [t_{n+1},\ \infty), \end{cases} \tag{1}$$

for $s \geq t$. Intuitively, Eq. (1) implies that, in addition to the constant discounting rate ρ, self n values the utility after the arrival of self $n+1$ by an extra discounting factor $\alpha \leq 1$. After the death of self n and the birth of self $n+1$, the agent uses the discount function $D_{n+1}(t,s)$ to value his utility.

Assuming agent is self n at time t and his lifetime utility over consumptions C is

$$\mathbb{E}\left[\int_t^{\infty} D_n(t,s)U(C(s))ds\right], \tag{2}$$

where $U(C) = \frac{C^{1-\gamma}}{1-\gamma}$.

The output has an AK production technology

$$Y(t) = AK(t), \tag{3}$$

where A is constant representing the productivity and capital stock K is the source of production.

The capital stock K evolves with the following process

$$dK(t) = \Phi(I(t), K(t))dt + \sigma K(t)dZ(t), \tag{4}$$

where σ is the volatility of capital depreciation, Z is a standard Brownian motion process, I denotes the investment, and adjustment cost $\Phi(I,K) = \phi(i)K$, where $i = I/K$ and $\phi(i) = i - \frac{\theta i^2}{2} - \delta$.

3. The benchmark with time-consistent preferences

In this section, we consider a benchmark economy in which agent has constant discount rates. The capital stock K is the only state variable in this economy, and thus the value function can be denoted by $V^*(K)$. The following proposition provides the equilibrium results for the case of time-consistent preferences.

Proposition 1. *If agent has time-consistent preferences, the value function is*

$$V^*(K) = \frac{1}{1-\gamma}(b^*K)^{1-\gamma}, \tag{5}$$

where $b^* = (A-i^*)^{\frac{\gamma}{\gamma-1}}\left(\phi'(i^*)\right)^{\frac{1}{\gamma-1}}$. *The optimal investment–capital ratio,* i^*, *solves the following equation*

$$A - i^* = \frac{1}{\phi'(i^*)}\left[\rho + (\gamma-1)\left(\phi(i^*) - \frac{\gamma\sigma^2}{2}\right)\right]. \tag{6}$$

The optimal consumption–capital ratio is $c^* = A - i^*$, *and Tobin's* q *is*

$$q^* = \frac{1}{\phi'(i^*)}. \tag{7}$$

The interest rate r^* *and the equity risk premium* rp^* *are*

$$r^* = \rho + \gamma\phi(i^*) - \frac{\gamma(\gamma+1)\sigma^2}{2}, \tag{8}$$

$$rp^* = \gamma\sigma^2. \tag{9}$$

It is easy to verify that the growth rates of consumption, investment, capital, and output are all equal. Therefore, after scaling by capital stock K, the consumption–capital ratio $c = C/K$, the investment–capital ratio $i = I/K$, and Tobin's q are all constant.

4. The naive agent with time-inconsistent preferences

In this section, we consider a case where agent is naive—the naive agent makes consumption–investment decisions under the wrong beliefs that the future selves act in the interest of the current self.[2] For example, starting with the self 0, the naive agent has the discount function $D_0(t,s)$, the future self 1 has the discount function $D_1(t,s)$, the future self 2 has the discount function $D_2(t,s)$, and so on. Since naive agent incorrectly believes that all future selves use the same discount function $D_0(t,s)$, he acts as if he can commit his future selves to behave as his current preference, and then the self 0 values the utility received after his death as $\alpha V^*(K)$, which is usually defined as the continuation value function.

We assume that the central planner maximizes the naive agent's utility with an infinite time horizon. Let $V_N(K)$ denote naive agent's value function, and I_N and C_N the optimal investment and consumption, respectively. By dynamic programming method, $V_N(K)$ solves the following Hamilton–Jacobi–Bellman (HJB) equation

$$\begin{aligned} \rho V_N(K) = \max_{I_N}\ & U(C_N) + \Phi(I_N, K)V_N'(K) + \frac{\sigma^2K^2}{2}V_N''(K) \\ & + \lambda(\alpha V^*(K) - V_N(K)), \end{aligned} \tag{10}$$

subject to the budget constraint $C_N + I_N = AK$. Using the first-order condition for investment I_N, we have

$$U'(C_N) = \Phi_{I_N}(I_N, K)V_N'(K). \tag{11}$$

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[2] This assumption on naivety is first proposed by Strotz (1956) and then used by Akerlof (1991) and O'Donoghue and Rabin (1999a,b), among others.