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Biasing selection contests with ex-ante identical agents

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HIGHLIGHTS

- We study contests where the designer wishes the winner to have high ability.
- A head start should be given randomly when the contestants are ex-ante identical.
- A head start should be given to the one who is more likely to have high ability.

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ABSTRACT

This note shows that when the designer of a contest wishes the winner have high ability, she is better off giving a head start to one of the contestants even if they are ex-ante identical. If the contestants are ex-ante asymmetric, the designer should give a head start to the one who is more likely to have high ability.

1. Introduction

In many cases of interest, the winner of a contest is expected to have higher ability than other contestants. Examples include architectural design competitions, competitive research grants, and recruitment and promotion of staff. In this note we demonstrate that when the objective of the designer of an all-pay contest is to select a contestant with high ability, the designer is better off giving an advantage to one of the contestants even if they are ex-ante identical from her viewpoint. We also show as a corollary that if the contestants are ex-ante asymmetric, the designer should give a head start to the one who is more likely to have high ability.

Selection in contests was first studied by Meyer (1991) who obtained related results to ours in a dynamic statistical (rather than game theoretic) model with two non-strategic agents. Unlike Meyer (1991), this note considers selection contests in a static one-shot game with fully strategic agents.[1] The economic insight from our approach is distinct and novel.[2] In particular, since our model is simple but features strategic agents, our insight could easily be contrasted to or incorporated into other models of all-pay contests.[3]

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1 In Meyer (1991) it is never optimal to bias a one-shot contest when the agents are ex-ante identical.

2 Other contributions to the literature on selection contests with strategic agents include Clark and Riis (2001) and Münster (2007) but they consider different design problems from ours and do not obtain unequal treatment of ex-ante identical agents.

3 Konrad (2009) offers an extensive overview on the effort maximizing contest literature. Recently Kirkegaard (2012) and Pérez-Castrillo and Wettstein (2013) have shown that unequal treatment of (ex-ante and ex-post) symmetric contestants may be beneficial in settings where the designer is concerned with the contestants' effort.

2. Model

Consider an all-pay contest with two agents, who have either high ability or low ability. While the agents know each other's ability, the principal cannot directly observe it. For expositional convenience we assume that one of the agents has high ability and the other has low ability. Our model can easily be modified to incorporate independent type distributions without changing the main insight.

Before the contest takes place, one of the agents is given a head start $a > 0$, and then each agent exerts effort $e_i \geq 0,\ i \in \{L, H\}$. In the contest, the one who generates the highest "score" is declared the winner and obtains a fixed reward V, while the reward for the other agent is normalized to 0.[4] The score of an agent is the sum of his effort and head start $a > 0$, if he has any. That is, the score of agent i is given by

$$s_i = \begin{cases} e_i + a & \text{if agent } i \text{ has head start} \\ e_i & \text{if agent } -i \text{ has head start.} \end{cases}$$

Each agent's winning probability $p(s_i, s_{-i}),\ i \in \{H, L\}$, is given by

$$p_i(s_i, s_{-i}) = \begin{cases} 1 & \text{if } s_i > s_{-i} \\ 0 & \text{if } s_i < s_{-i} \\ p_i \in [0, 1] & \text{if } s_i = s_{-i}. \end{cases}$$

We assume that the effort cost is lower for the high ability agent. In particular, denoting $c_H(\cdot)$ the effort cost function of the agent with high ability and $c_L(\cdot)$ that of the agent with low ability, we assume that $c_i(0) = 0, c_i'(\cdot) > 0, c_i''(\cdot) \geq 0$ and $c_H(\cdot) \leq c_L(\cdot)$ for any given effort level.[5] We further assume $c_H'''(\cdot) \leq 0$, that is, the third derivative of the cost function of the high ability agent is non-positive, which covers e.g., linear and quadratic effort costs. We will discuss this assumption when interpreting our result later. The payoff function of the agent with high ability and that of the agent with low ability are given respectively by

$$u^H = \begin{cases} V - c_H(e_H) & \text{if the agent wins} \\ -c_H(e_H) & \text{if the agent loses;} \end{cases} \quad \text{and}$$

$$u^L = \begin{cases} V - c_L(e_L) & \text{if the agent wins} \\ -c_L(e_L) & \text{if the agent loses.} \end{cases}$$

The objective of the principal/contest designer is to maximize the probability that the agent with high ability wins the contest. Note that the head start is given by the principal to a particular agent ex-ante. Since she is not informed of each agent's type, she cannot be certain whether she has given the head start to the low or high ability agent.

Throughout this note we will refer as an illustration to the case of linear costs, where

$$c_H(e_H) = e_H \quad \text{and} \quad c(e_L) = ce_L \quad \text{with } c > 1. \tag{1}$$

Definition. The *selection power* of a contest is the probability that the high ability agent achieves a higher score in equilibrium.

3. Bias for selection

In this section we first derive the equilibrium effort without head start ($a = 0$) as a benchmark. We then compute the equilibrium effort when the agent with high ability receives the head start $a > 0$, and also when the agent with low ability receives the head start. Lastly we study the selection power of the contest from the viewpoint of the principal who is uncertain about the ability of the agents, and prove our main result.

3.1. Unbiased contest ($a = 0$)

In this case, an agent's score represents his effort. Following Siegel (2009), in equilibrium the effort density of the low ability agent and the high ability agent is given by

$$f_L(e_L) = \frac{1}{V} c_H'(e_L) \quad \text{for } e_L \in \left(0, c_L^{-1}(V)\right] \text{ with mass}$$
$$1 - \frac{1}{V} c_H(c_L^{-1}(V)) \text{ at } e_L = 0; \text{ and}$$
$$f_H(e_H) = \frac{1}{V} c_L'(e_H) \quad \text{for } e_H \in \left[0, c_L^{-1}(V)\right],$$

respectively. The winning probability of the high ability agent in this neutral contest is given by

$$\begin{aligned} P_N &= \int_0^{c_L^{-1}(V)} f_H(e_H) F_L(e_H) de_H \\ &= \int_0^{c_L^{-1}(V)} \frac{1}{V} c_L'(e_H) \left[\frac{1}{V} c_H(e_H) + 1 - \frac{1}{V} c_H(c_L^{-1}(V))\right] \\ &= 1 - \frac{1}{V} c_H(c_L^{-1}(V)) + \frac{1}{V^2} \int_0^{c_L^{-1}(V)} c_L'(e_H) c_H(e_H) de_H. \end{aligned} \tag{2}$$

It is easy to check that, under the linear costs in (1), the probability of winning is given by

$$P_N = 1 - \frac{1}{2c}. \tag{3}$$

3.2. Bias for high ability agent

Let us consider the case in which the high ability agent receives the head start, namely $s_H = a + e_H$. Note that the "score density" of each agent for the competing range is the same as above. In Siegel (2009)'s terms the high ability agent's "reach" is higher, and thus we know that the low ability agent obtains the expected payoff of 0, which also means his highest effort is $c_L^{-1}(V)$. It follows that in equilibrium,

$$f_L(e_L) = \frac{1}{V} c_H'(e_L - a) \quad \text{for } e_L \in \left(a, c_L^{-1}(V)\right] \text{ with mass}$$
$$1 - \frac{1}{V} c_H(c_L^{-1}(V) - a) \text{ at } e_L = 0; \text{ and}$$
$$f_H(e_H) = \frac{1}{V} c_L'(e_H + a) \quad \text{for } e_H \in \left(0, c_L^{-1}(V) - a\right] \text{ with mass}$$
$$\frac{1}{V} c_L(a) \text{ at } e_H = 0.$$

The winning probability of the high ability agent is given by

$$\begin{aligned} P_H(a) &= \int_0^{c_L^{-1}(V)-a} f_H(e_H) F_L(e_H + a) de_H \\ &\quad + \frac{1}{V} c_L(a) \left(1 - \frac{1}{V} c_H(c_L^{-1}(V) - a)\right) \\ &= \int_0^{c_L^{-1}(V)-a} \frac{1}{V} c_L'(e_H) \left[\frac{1}{V} c_H(e_H)\right. \end{aligned}$$

[4] In this note we take V and a as exogenous variables, in order to highlight whether to use a head start, and if so, which agent should receive it. In reality the level of the reward and that of the head start may indeed be difficult for the contest designer to control. The reward may involve non-monetary components such as honour and prestige that cannot be fine-tuned. The judge of a contest who exercises favouritism (head start) for a contestant may not be the same person as the designer, in which case it may be impossible to control the size of the head start precisely.

[5] Instead of convex effort costs we can equivalently assume a concave score function with linear effort costs.