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A bottom poor sensitive Gini coefficient and maximum entropy estimation of income distributions

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ABSTRACT

A bottom poor sensitive Gini coefficient (pgini) is defined by replacing income observations with their reciprocal values in the Gini coefficient. The underlying true income share function can be derived approximately using the maximum entropy method given the pgini coefficient.

1. Introduction

The original Gini is known to be less sensitive to the small income changes of the poorest group. A decrease in the Gini may not necessarily be an improvement for the poorest group. Bonferroni (1930) and Ryu (2008) modified the definition of the Gini to give heavier weight to the poor group. Atkinson (1970) suggested an index of inequality to measure inequality under different value judgments where the zero value represents the indifference to inequality and infinity represents the Rawlsian criterion. Yitzhaki (1983) extended the Gini to reflect a preference for inequality where the aversion to inequality rises as ν goes from 0 to infinity. Kakwani (1980) and Donaldson and Weymark (1983) developed versions of the extended Gini that depend on social welfare functions. Wolfson (1994) introduced a polarization index to check the collapse of the middle class and their movements to either the poor class or rich class. Small income changes of the poorest group may (or may not) be well described by the above indices.

This paper defines an inconvenience level with the reciprocal value of income. Higher income provides more convenience and a smaller income gives a severe inconvenience. The rich can hire help and use superior facilities to save time and effort; in addition, a society has a total amount of inconveniences that are unevenly distributed to individuals. The inequality of inconvenience levels is measured with the Gini coefficient and is found to be highly correlated with the income shares of the bottom 5% poorest group. The correlation coefficient is 0.994 for U.S. family income shares. This inequality of inconvenience level measurement is called the bottom poor sensitive Gini coefficient (pgini) in this paper. The inconvenience Lorenz curve is defined with the sum of inconvenience shares and the pgini can be derived with the ellipse area above the Lorenz curve. Other explanations based on the original Gini coefficient and the Lorenz curve can be replicated for the pgini case. Dagum (1997) showed that this could be decomposed into several parts, the contribution of inequality within groups and the contribution of inequality between groups. The poverty line can also be drawn in the inconvenience Lorenz curve.

Section 2 defines the pgini. Section 3 derives the unknown income share function approximately. Section 4 compares the performance of the pgini with other measures. Section 5 provides the conclusion.

2. Definition of pgini

$$\text{pgini} \equiv \frac{1}{2n^2\mu}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|\frac{1}{y_i}-\frac{1}{y_j}\right|. \quad (1)$$

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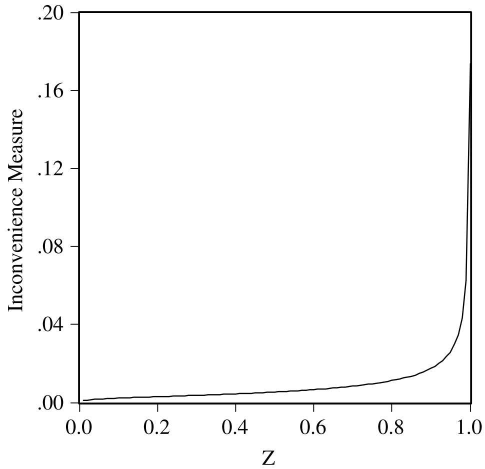

Fig. 1. Inconvenience measure.

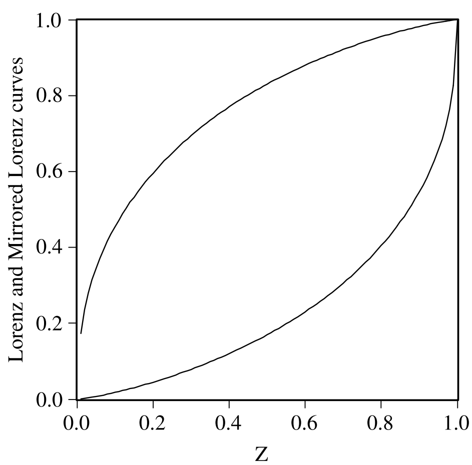

Fig. 2. Inconvenience Lorenz curve.

A society has a total amount of inconveniences, $\sum_{i=1}^{n}(1/y_i)$ and the mean inconvenience is $\mu = (1/n)\ \sum_{i=1}^{n}(1/y_i)$. Each individual has inconvenience share r_i and pgini can be rewritten as

$$r_i = \frac{1}{n\mu}\frac{1}{y_i} \tag{2}$$

$$\text{pgini} \equiv \frac{1}{2n}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|r_i - r_j\right|. \tag{3}$$

It can be shown that the pgini satisfies the necessary conditions required for an inequality measure stated in Fields and Fei (1978), the scale of irrelevance, symmetry, and rank-preserving equalization.

The inconvenience Lorenz curve is defined with the sum of inconvenience shares. The richest person with least inconvenience is located at $z = 0$ and the poorest person with the most inconvenience is located at $z = 1$ (see Fig. 1).

The pgini is equal to the ellipse area of Fig. 2. Other inequality measures assign different weights to the poor and rich individuals. The Bonferroni index is defined as

$$BI = 1 - \int_0^1 \frac{L(z)}{z}dz$$

where $L(z)$ is the original Lorenz curve. Bonferroni (1930) and Ryu (2008) put heavier weights to the poor income groups. For discrete observations,

$$BI = \frac{1}{n-1}\sum_{i=1}^{n-1}\frac{P_i - Q_i}{P_i} \quad \text{where } P_i = \frac{i}{n} \text{ and } Q_i = \sum_{j=1}^{i}\frac{x_j}{n\gamma} \tag{4}$$

where γ is the mean income and x_j is the observed income of the jth individual.

Wolfson (1994) introduced a scalar polarization index to show an insufficient explanation of the Gini for certain income changes. The lowered Gini coefficient is not necessarily a desirable consequence if the middle class is eliminated from society. The scalar polarization index is

$$P = \frac{4\gamma}{m}\left|0.5 - L(0.5) - 0.5\text{Gini}\right| \tag{5}$$

where m is the median income and γ is the mean income. The Wolfson polarization index is also used to compare and check its sensitivity with the income share changes of the poor group.

3. Maximum entropy estimation of income shares from the pgini

We show that the knowledge of pgini is equivalent to the knowledge of the first moment of inconvenience shares. The Lorenz curve for the inconvenience measure is defined as

$$PL \equiv \int_0^z r(z')dz'$$

where $r(z)$ is the inconvenience share function and the coordinate z is the population coordinate with $z = 0$ for the richest person with the least inconvenience and $z = 1$ for the poorest person with the largest inconvenience. Consider the partial integration of

$$\int_0^1 zdPL = zPL(z)_0^1 - \int_0^1 PL(z)dz = 1 - g$$

where $g \equiv \int_0^1 PL(z)dz = \frac{1-\text{pgini}}{2}$.

Since

$$dPL(z) = r(z)dz.$$

The mean of the inconvenience share function is

$$\mu_1 = \int_0^1 zr(z)dz = 1 - g = \frac{1+\text{pgini}}{2}. \tag{6}$$

Knowledge of the pgini is equivalent to the knowledge of the first moment of the true inconvenience share function. A similar result is also reported in Lerman and Yitzhaki (1984) for the original Gini coefficient.

The inconvenience share distribution can be derived from the given first moment. Solving an entropy maximization problem as stated in Ryu (1993)

$$\text{Max}_r W \equiv -\int r(z)\log r(z)dz \tag{7}$$

satisfying

$$\int zr(z)dz = \mu_1, \tag{8}$$

the Lagrangian method produces

$$r(z) = \exp\left[a + bz\right] = \left[\frac{b}{e^b - 1}\right]\exp[bz] \tag{9}$$

where the normalization condition of the share function is used to remove a. Now the first moment condition (8) produces,

$$\mu_1 = \left[\frac{b}{e^b - 1}\right]\int_0^1 z\exp[bz]dz = \frac{1+\text{pgini}}{2}. \tag{10}$$