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Bayesian calibration and number of jump components in electricity spot price models

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ABSTRACT

We find empirical evidence that mean-reverting jump processes are not statistically adequate to model electricity spot price spikes but independent, signed sums of such processes are statistically adequate. Further we demonstrate a change in the composition of these sums after a major economic event. This is achieved by developing a Markov Chain Monte Carlo (MCMC) procedure for Bayesian model calibration and a Bayesian assessment of model adequacy (posterior predictive checking). In particular we determine the number of signed mean-reverting jump components required in the APXUK and EEX markets, in time periods both before and after the recent global financial crises. Statistically, consistent structural changes occur across both markets, with a reduction of the intensity and size, or the disappearance, of positive price spikes in the later period. All code and data are provided to enable replication of results.

1. Introduction

Electricity spot markets have multiple fundamental drivers, for example baseload and renewable production (Würzburg et al., 2013). Disturbances in these drivers, such as plant outages and renewable gluts, can clearly have different dynamic characteristics and consequences. Since sharp disturbances create spikes in electricity spot prices (Seifert and Uhrig-Homburg, 2007) we may hypothesise that, over time, disturbances in different drivers give rise to spikes with statistically distinguishable directions, frequencies, height distributions and rates of decay. It has recently been demonstrated that electricity spot price formation can evolve over time (Brunner, 2014). Thus we may also hypothesise that the statistical characteristics of electricity price spikes will evolve in step with underlying economic events and developments, such as shifts in demand and increasing renewable penetrations.

In this paper we find empirical support for these two hypotheses. To this end we use *multi-factor* electricity spot price models, with multiple superposed mean-reverting components and a seasonal trend (Benth et al., 2007). This allows statistical patterns such as mean reversion, seasonality and spikes to be reproduced in modelling. Crucially for the present study, this approach also allows the statistical modelling of multiple spike components with differing frequencies, height distributions, decay rates, and directions (positive or negative). We demonstrate that in some electricity markets two types of positive spike are observed, while other markets require the inclusion of negative spikes. The modelling of negative spikes is an area of emerging interest (Fanone et al., 2013) as renewable penetrations, and hence gluts in renewable production, increase. Finally we document evolution of the statistical spike structure through periods of economic change by comparing two markets across two

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time periods, one before the recent global financial crises (2000–2007) and another afterwards (2011–2015) and reflect on possible interpretations of the results.

The calibration of multi-factor models is a highly challenging task and existing approaches typically involve making strong *a priori* assumptions, such as setting thresholds for jump sizes, which may mask the true statistical structure. Methodologically, we develop a Bayesian approach to calibration based on Markov Chain Monte Carlo (MCMC) methods. This goes beyond previous work by making minimal assumptions and enables us, for example, to estimate models with multiple spike components acting in the same direction, a feature which is confirmed empirically (in 2001–2006 data from the APXUK electricity spot market). In order to assess the number of mean-reverting jump components required we perform a Bayesian procedure of *posterior predictive checking*.

1.1. Background and related work

Econometric models of electricity spot prices have a number of important applications. They provide stochastic models which can be used by traders to analyse financial options on power (Benth et al., 2007), and by power system planners to conduct real options analyses for flexible physical assets such as storage and cogeneration (Moriarty and Palczewski, 2017; Kitapbayev et al., 2015). Further, the pronounced price spikes which characterise spot electricity markets are of central interest to electricity market regulators who monitor and influence the economics of markets, aiming for example to prevent perceived abuses of market power (Stephenson and Paun, 2001).

The complexity of electricity spot price models, and multi-factor models in particular, makes their analysis statistically challenging and has given rise to a substantial literature. A single-factor model including the above stylised features was introduced by Clewlow and Strickland (2000). Through the use of a threshold, the single-factor model of Geman and Roncoroni (2006) incorporates two jump regimes: when the price is below the threshold jumps are positive, and when the price exceeds the threshold jumps are negative. Beginning with Lucia and Schwartz (2002) multi-factor models have expressed the price as a sum of unobservable or *latent* processes (*factors*) with distinct purposes, for example the modelling of short-term and long-term price variations respectively. Unlike many single factor models, multi-factor models do not imply a perfect correlation between changes in spot, future and forward prices, which is consistent with the non-storability of electricity (Benth and Meyer-Brandis, 2009). The model of Lucia and Schwartz (2002) has two factors, namely a Gaussian mean-reverting process and an arithmetic Brownian motion (that is, a scaled Brownian motion with drift). Interestingly, while also developing a two-factor model, Seifert and Uhrig-Homburg (2007) explicitly refer to the physical origins of various types of jumps. Beyond two-factor models, a simple and flexible multi-factor model with jumps is given in Benth et al. (2007). Estimation procedures for this model are discussed in Meyer-Brandis and Tankov (2008), although the latter work adds strong assumptions in order to obtain tractable methods.

The interdependency between parameters in multi-factor models, in particular, is a challenge to calibration methods. A straightforward approach is to first separate the observed values into factors using signal filtering techniques, in order to subsequently employ classical maximum likelihood estimation. Such methods effectively assume that some of these interdependencies may be neglected, and this approach is taken for example in Meyer-Brandis and Tankov (2008) and Benth et al. (2012). An alternative is the joint estimation of latent factors, for which there are two leading methodologies in the literature: *expectation-maximisation (EM)* and *Markov Chain Monte Carlo (MCMC)* methods. While EM produces point estimates for parameters in either a Bayesian or frequentist framework[1] (see, for example, Rydén et al., 2008), MCMC is able to generate samples from posterior parameter distributions. Particularly in models with multiple parameters and latent processes, these interdependencies may result in likelihood surfaces and posterior distributions which are rather flat around their maxima. While EM suffers from Monte Carlo errors which amplify the usual difficulties in numerical optimisation for such problems, MCMC estimates the posterior distribution providing an analyst with a more complete picture of the interrelations between parameters.

In related contexts, MCMC has been applied to fit continuous-time stochastic volatility models to financial time series, where the price is a diffusion process whose volatility is a latent mean reverting jump process or the sum of a number of such processes (called a superposition model). In this line of research a missing data methodology is employed whereby the observed process is augmented with one or more latent marked Poisson processes and the MCMC procedure generates posterior samples in this high dimensional augmented state space. Examples include Roberts et al. (2004), Griffin and Steel (2006) and Frühwirth-Schnatter and Sögner (2009). Since energy prices additionally exhibit jumps directly in their paths, MCMC has been applied to extensions of these models in which a diffusion process with stochastic volatility is superposed with a jump process, see Green and Nossman (2008) in the context of electricity and Brix (2015) for gas prices. Technically the latter two papers estimate a discrete approximation of the models whereas in this study we pursue *exact* inference for continuous time models.

1.2. Contribution

From the modelling point of view, a novelty of the present study is that the price is a superposition of more than one jump component, each with its own sign, frequency, size distribution and decay rate, along with a diffusion component. This approach acknowledges that the negative price spikes attributable to rapid wind power fluctuations may, for example, have quicker decay than the infrequent larger positive spikes due to major disturbances such as outages of a traditional generation plant. The inclusion of multiple jump components also addresses the following problem identified in Green and Nossman (2008) and Brix (2015). In two-factor models, jumps of intermediate size must be accounted for either in the diffusion process (forcing unlikely spikes in the Brownian motion path) or the jump process (implying additional jumps). While the former can lead to an overestimation of volatility in the diffusion process, the latter may result in an overestimation of the intensity of the jump process, which is independent of the jump sizes. The inclusion of a second jump process with its own mean jump size and rate of mean reversion removes this dichotomy, offering an alternative to the inclusion of stochastic volatility in the diffusion process.

Our first methodological contribution is an MCMC algorithm for *exact* Bayesian inference on superposed OU models with diffusion and multiple jump components. We contrast exact inference with a commonly used estimation procedure using a discrete time model which is an approximation to continuous dynamics. While this approximation is often used for practical reasons including simplified and/or tractable implementation, it is not possible to assess *a priori* the extent of the estimation error introduced by the approximation

[1] Two possible approaches to the calibration of model parameters are commonly referred to as *frequentist* and *Bayesian*. In the frequentist approach, one seeks to derive point estimates of 'true' parameter values from the data, for example by finding the maximiser of a likelihood function. An alternative viewpoint is taken in the Bayesian approach, where the unknown parameters are first assigned a probability *distribution* representing prior beliefs about their value. This prior distribution is combined with the observed data to produce an updated probability distribution representing the posterior beliefs about the parameters given both the prior and the data.