Price dynamics, social networks and communication

Bingqing Li [a,*], Lijia Wang [a], Guoxiang Lu [b]

[a] *School of Finance, Nankai University, No. 38 Tongyan Road, Tianjin, China*
[b] *School of Statistics and Mathematics, Zhongnan University of Economics and Law, No. 182 Nanhu Avenue, Wuhan, China*

A R T I C L E   I N F O

A B S T R A C T

A stock price dynamics model is developed in consideration of social network communication in financial markets. Considering a nonlinear feedback effect of price returns, we establish a self-organising system of price dynamics. Results show that the movement of prices depends on the topologies of networks and the communication effect. Furthermore, the self-reinforcing feature of price dynamics is explored and bubbles and crashes are explained as alternate strong positive and negative self-reinforcing processes of prices.

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1. Introduction

In economic communities, interpersonal communication is important in the diffusion of information (Shiller, 1995). Surveys by Shiller and Pound (1989) indicated that direct informal communication is very important in investor decisions. Hong et al. (2005) and Pool et al. (2015) both found that the holdings and trades of fund managers who have social connections were correlated. Ivković and Weisbenner (2007) also found a strong relationship between households and their neighbors in stock purchases. Moreover, Cohen et al. (2008) provided evidence that education networks connections between fund managers and corporate board members had a great effect on portfolio performance. Given these studies, the process of communication and information dissemination can be well captured by a social network, which plays a pivotal role in financial markets.

Recently, some studies demonstrated that the topologies of agents' social networks have had a significant effect on information diffusion and agents' behavior and thus on price dynamics. Hein et al. (2012) found that price volatility increased when the level of network centralization was raised. Panchenko et al. (2013) indicated that network topologies influenced the stability of asset price dynamics. However, these studies mostly focused on simulation results which is difficult to analyze theoretically. In this paper, we try to establish a theoretical price dynamics model by applying the susceptible-infected-susceptible (SIS) epidemic model (Mieghem et al., 2009). We analytically investigate how network topologies influence stock prices in consideration of social network communication.

It was assumed that price dynamics were driven by decisions of boundedly rational and interacting agents. Communication takes place through social networks based on which agents are spatially heterogeneous. Through social network

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* Corresponding author.
  E-mail addresses: libq@nankai.edu.cn, bqlink@126.com (B. Li), ljwnku@163.com (L. Wang), lgxmath@zuel.edu.cn (G. Lu).

communication, agents' behavior is influenced by others. We describe the transition of agents' holding states by the SIS model. Considering a nonlinear feedback effect of price returns on the communication effect, a self-organising system of price dynamics is developed. We derive the conditions under which the price increases or decreases for regular social networks. In addition, the movement of prices depends on the steady price which is closely related with topologies of networks and the communication effect. Further, the self-reinforcing feature of price dynamics is explored and bubbles and crashes are explained as alternate strong positive and negative self-reinforcing processes of prices.

The main contribution of this paper is that we develop a price dynamics model considering social network communication and analytically show that the topologies of networks influence stock prices. In the rest of our paper, we introduced the model (Section 2), presented analytical (Section 3) and numerical results (Section 4), and concluded (Section 5).

2. Model

Following work by Lux (1995), this study assumed two groups of agents exist in the economy. Agents in the first group are connected by social networks. The other group of agents can be seen as a counterparty to the first one, providing liquidity to the market. We assumed the price was driven by the agents' behavior of the first group, so the following discussion focused on the agents in the first group.

Let $S(t) = \{s_1(t), s_2(t), \ldots, s_N(t)\}$ represents the holding states of agents at time t,

$$s_i(t) = \begin{cases} 1 & \text{agent } i \text{ holds a stock and is called } \textbf{holder}; \\ 0 & \text{agent } i \text{ does not hold a stock and is called } \textbf{unholder}. \end{cases}$$

There are N agents connected in a network G in which nodes represent agents, and edges are the connective links between them.[1] Let $A = (a_{ij})_{n \times n}$ be the adjacent matrix of the N-agent network where $a_{ij} = 1$ if there is a link between agent i and j; otherwise $a_{ij} = 0$. Each agent communicates with the set of neighboring agents connected to it, denoted by $N_i(G) = \{j | a_{ij} = 1\}$. The number of neighbors is called the degree, defined as $d_i = \sharp\{j | j \in N_i(G)\}$ for agent i. The spectral radius of network G is $\rho_A = max\{|\lambda_i|, i = 1, 2, \ldots, N\}$, in which λ_i is the eigenvalue of matrix A.

It was assumed that all agents were initially endowed with the same capital which can trade only one unit of stock. Constraints on selling short or buying long were imposed. Similar to Lux (1995) and Panchenko et al. (2013), we assumed that agents' decisions were influenced by information from others. The changing decisions influenced by others seems like the contagion of a disease, thus we applied the SIS model to describe the transition of agents' holding states. Assume the buying decision of an unholder was affected by the information of his or her holding neighbors and a holder's selling decision was influenced by the market information directly. Specifically, an unholder would be infected to be a holder with rate $\varphi(t)$ and a holder would become an unholder with rate $\theta(t)$. A feedback effect of historical return $R(t)$ on $\varphi(t)$ and $\theta(t)$ was considered, and (1) $0 < \varphi(t) < 1$ and $0 < \theta(t) < 1$; (2) $\frac{\partial \varphi(t)}{\partial R(t)} > 0$, $\frac{\partial \theta(t)}{\partial R(t)} < 0$. Let $\tau(t) = \frac{\varphi(t)}{\theta(t)}$ be known as the **communication effect** and $\frac{\partial \tau(t)}{\partial R(t)} > 0$.

Let $\pi_i(t) = Prob\{s_i(t) = 1\}$. This denotes the probability of agent i holding a stock at time t. The transition of holding states can be modeled by a Markov process. For any sufficiently small Δt, there is

$$\pi_i(t + \Delta t) = \begin{cases} \varphi(t) n_i(t) \Delta t, & \text{if } s_i(t) = 0 \\ 1 - \theta(t) \Delta t, & \text{if } s_i(t) = 1 \end{cases}$$

where $n_i(t)$ is the number of holders connected with agent i at time t, denoted by $n_i(t) = \sharp\{s_j(t) = 1 | j \in N_i(G)\}$.

Let $\Delta t \to 0$, we have

$$\frac{d\pi_i(t)}{dt} = \varphi(t) n_i(t)(1 - \pi_i(t)) - \theta(t)\pi_i(t) \tag{1}$$

Assume that price dynamics were driven by the stock demand of agents, which can be expressed as

$$d(P(t)) = \beta d(\eta(t)), \beta > 0. \tag{2}$$

where $\eta(t) = \Sigma_{i=1}^{N} s_i(t)$ is the total number of holders at time t, and β is an adjustment coefficient.

3. Analytical results

For a constant communication effect τ, if there exists a moment T, for $t > T$ satisfy

$$\frac{d\pi_i(t)}{dt} = 0, i = 1, 2, \ldots, N. \tag{3}$$

then the market reaches a steady state, in which the holding probability of each holder remains unchanged. The expectation of stock price in the steady state was named the **steady price** and was denoted by P_τ^s. Note that, there is a steady price corresponding to the communication effect at each time. Specifically, the expected price corresponding to the communication effect $\tau(t)$ at time t was denoted by $P_{\tau(t)}^s$.

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[1] Communication was assumed to be reciprocal, so connections were undirected.