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Forecasting volatility with interacting multiple models

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ABSTRACT

We examine the performance of Kalman filter techniques in forecasting volatility. We find that the simple implementation of an online Kalman filtering procedure that combines commonly used forecasting models with market-based estimates improves the accuracy of volatility forecasts. Furthermore, we demonstrate that the Interacting Multiple Model algorithm, which combines multiple Kalman filters, provides the most accurate volatility forecasts overall.

1. Introduction

The past two decades have spawned vast literature on volatility estimation and forecasting, ranging from time series models to market-based estimates implied from options. However, with such a large range of estimation techniques at their fingertips, academics and practitioners alike are often overwhelmed by the paradox of choice. Given the potentially incorrect assumptions regarding volatility dynamics in model-based estimates (Nelson, 1992; Nelson and Foster, 1995) and trading noise and market irrationalities in aggregate market-based estimates (Hentschel, 2003), practitioners increasingly combine forecasts to improve the accuracy of predictions (see, for example, Timmermann, 2006). However, frequently the forecast weighting algorithms are chosen arbitrarily and lack optimality and sophistication.

The problem of multiple model estimation has been addressed in other fields through the use of online, or real-time, Kalman filtering. The Kalman (1960) filter optimally combines multiple model estimates to produce forecasts with lower errors than the two constituent estimates. While commonly applied to solve control engineering problems, most notably object tracking, the application of online Kalman filtering to financial data remains relatively unexplored. In contrast to engineering, where systems are generally deterministic, in financial and econometric applications, parameter specifications are not necessarily known (Harvey, 1989; Wells, 1996). Consequently, the financial literature has evolved mainly around quasi-maximum likelihood parameter estimation where the filter is effectively reverse engineered using a set of actual data to uncover the system matrices. This model fitting application has been demonstrated in a wide range of studies, including the estimation of Stochastic Volatility Models (Hwang and Satchell, 2000; Jacquier et al., 1994), beta (Wells, 1996; Choudhry and Wu, 2008), Australian interest rates (Bhar, 1996) and the currencies of emerging markets (Wang and Wong, 1997).

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However, the Kalman filter is more powerful than a simple model fitting tool. We follow the work of Athans (1974) and combine model and market-based volatility estimates using the online filtering algorithm to increase the overall precision of forecasts. Although the Kalman filter can only combine a single model with a single sensor estimate the development of multiple model adaptive estimation (MMAE) techniques such as Bayesian switching, as outlined in (Simon, 2006), and more complex Interacting Multiple Model (IMM) methods, as presented in Li (1992) and Sworder and Boyd (1999) can employ banks of Kalman filters in parallel, each running a different model which are then fused to obtain a more accurate estimate of volatility. The IMM Kalman filter allows volatility dynamics to be represented by a number of models and thus provides a powerful framework for the estimation of volatility that transitions between a set of dynamics.

The closest relative of the broader multiple model class of estimation methods in finance is the dynamic Markov regime switching models introduced by Hamilton (1989). The Markov regime switching involves swapping out parameters within a model based on different sample periods. However, based on the presentation in Kim and Nelson (1999), the model itself remains the same across all regimes. As such the behavioural assumptions regarding the dynamics of the model are held through the regime switch. This is in direct contrast to the broader MMAE methods which allow multiple models with differing dynamics to compete. These types of Markov regime switching models have been used to describe stock returns (Schaller and Van Norden, 1997), option pricing (Bollen, 1998; Buffington and Elliott, 2002), long run volatility estimation (Calvet and Fisher, 2004) and even probabilities of financial crises (Coe, 2002).

2. Methodology and data

2.1. The Kalman filter implementation

The Kalman filter addresses the general problem of trying to estimate the state of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x_t = Ax_{t-1} + Bu_{t-1} + \varepsilon_{t-1}, \quad \varepsilon_{t-1} \sim \mathcal{N}(0, Q) \tag{1}$$

with a measurement equation

$$y_t = Cx_t + \ \eta_t, \quad \eta_t \sim \mathcal{N}(0, R) \tag{2}$$

where A, B and C are the state transition, control and observation matrix, respectively. ε_{t-1} and η_t are random variables representing the process and measurement noise, respectively, assumed to be independent of each other with Gaussian white processes.[1]

To apply the Kalman filter we express the underlying volatility processes in linear state-space form. In our model, the state vector x_t represents the daily variance rate and y_t provides direct observation of the market-based volatility estimate implied from the options market (IVOL). The process noise ε_{t-1} reflects the model error while the sensor noise η_t reflects trading noise in the options market. The model error and the trading noise covariance matrix are given by Q and R, respectively. For consistency, we fix R across all Kalman filters to the variance of the IVOL series. In multi-state systems we assume an equal process noise variance across all states and independence across the error terms.

2.2. Interacting Multiple Model (IMM) estimator

The IMM algorithm is a technique for combining the output estimates from multiple Kalman filters in parallel with each filter describing a different dynamic model of volatility. The estimation algorithm can be separated into four steps.[2] In the first step, the state estimate and covariance for each model from the previous cycle are mixed using a set of conditional model probabilities computed from the previous update. These mixed state $\bar{x}^{(i)}_{t-1|t-1}$, and covariance $\bar{P}^{(i)}_{t-1|t-1}$, estimates are the inputs of the Kalman filter at time t as they provide the best possible estimate of all information at time t.

$$\bar{x}^{(i)}_{t-1|t-1} = \sum_j \hat{x}^{(j)}_{t-1|t-1}\mu^{j|i}_{t-1} \tag{3}$$

$$\bar{P}^{(i)}_{t-1|t-1} = \sum_j \left[P^{(j)}_{t-1|t-1} + \left(\bar{x}^{(i)}_{t-1|t-1} - \hat{x}^{(j)}_{t-1|t-1}\right)\left(\bar{x}^{(i)}_{t-1|t-1} - \hat{x}^{(j)}_{t-1|t-1}\right)^T \right]\mu^{j|i}_{t-1} \tag{4}$$

where $\hat{x}^{(j)}_{t-1|t-1}$ and $P^{(j)}_{t-1|t-1}$ are the updated state and covariance, respectively, from the j^{th} Kalman filter from the previous time step (*t-1*). The conditional probability of the volatility in the i^{th} model state, which transited from the j^{th} model state $\mu^{j|i}_{t-1}$ is given by

$$\mu^{j|i}_{t-1} = \frac{\pi_{ji}\mu^{(j)}_{t-1}}{\sum_j \pi_{ji}\mu^{(j)}_{t-1}} \tag{5}$$

[1] Several pieces of literature provide a thorough derivation of the Kalman filter and describe its optimality in linear systems. A particularly simple and intuitive outline of the algorithm is provided by (Welch & Bishop, 2006).

[2] For a full derivation of the IMM estimator see Li (1992).