A parallel code to calculate rate-state seismicity evolution induced by time dependent, heterogeneous Coulomb stress changes

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A B S T R A C T

The estimation of space and time-dependent earthquake probabilities, including aftershock sequences, has received increased attention in recent years, and Operational Earthquake Forecasting systems are currently being implemented in various countries. Physics based earthquake forecasting models compute time dependent earthquake rates based on Coulomb stress changes, coupled with seismicity evolution laws derived from rate-state friction. While early implementations of such models typically performed poorly compared to statistical models, recent studies indicate that significant performance improvements can be achieved by considering the spatial heterogeneity of the stress field and secondary sources of stress. However, the major drawback of these methods is a rapid increase in computational costs. Here we present a code to calculate seismicity induced by time dependent stress changes. An important feature of the code is the possibility to include aleatoric uncertainties due to the existence of multiple receiver faults and to the finite grid size, as well epistemic uncertainties due to the choice of input slip model. To compensate for the growth in computational requirements, we have parallelized the code for shared memory systems (using OpenMP) and distributed memory systems (using MPI). Performance tests indicate that these parallelization strategies lead to a significant speedup for problems with different degrees of complexity, ranging from those which can be solved on standard multicore desktop computers, to those requiring a small cluster, to a large simulation that can be run using up to 1500 cores.

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1. Introduction

A common approach to modeling aftershocks and seismic swarms consists of calculating Coulomb stress changes, coupled with the time-dependent frictional response described by the Dieterich constitutive law (Dieterich, 1994). These models are known as Coulomb-rate-state (CRS) models. Most previous CRS models use a single value of stress at each cell: spatial variations and temporal variations induced by postseismic processes are not taken into account. Previous studies (Marsan, 2006; Helmstetter and Shaw, 2009; Hainzl et al., 2009; Cattania et al., 2014) indicate that the heterogeneity of the stress field has a profound effect on both the spatial and temporal distribution of seismicity. Stress heterogeneity is due to several factors, including elastic heterogeneity from different rock types or fluid content, small scale distribution of slip on the mainshock fault, the existence of multiple receiver faults, and the presence of stress gradients within each cell due to their finite size. Large uncertainties also exist in the input data: in particular, uncertainties in slip models lead to large errors in Coulomb stress calculations (e.g. Hainzl et al. (2009) and Cattania et al. (2014)).

We present a parallel code written in C, which implements a CRS model to calculate seismicity evolution from seismic or aseismic (time dependent) sources. The software allows to estimate aleatoric uncertainties (stress field heterogeneity) due to multiple receiver faults and to the finite grid size, and to propagate epistemic uncertainties (due to alternative slip models) which can be used to quantify the uncertainty of the forecast. Modeling time dependent effects and uncertainties leads to a rapid growth in computational requirements, which limits the model resolution and domain size. We overcome these restrictions by implementing two layers of parallelization, for shared and distributed memory systems. The paper is structured as follows: we first describe the theoretical framework and the model implementation; we then outline the optimization strategies, and in particular parallelization and its scaling with number of threads. Finally, we provide usability tips and discuss the potential uses of the software.

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2. Theoretical framework

2.1. Coulomb stress changes

According to the Coulomb–Mohr criterion, failure on a plane is promoted by an increase in shear stress (τ) and inhibited by an increase in normal stress (σ_{tot}). Therefore, a useful quantity to predict how a change in the local stressing state affects seismicity is the Coulomb stress change, defined as

$$\Delta CFS = \Delta\tau - \mu\Delta\sigma_{tot} \tag{1}$$

with μ the coefficient of friction. The total normal pressure on the fault, σ_{tot}, is given by $\sigma_{tot} = \sigma - p$, where σ is the externally applied normal pressure and p is the pore pressure. Different models exist to describe how pore pressure changes in response to applied stresses; CRS employs the apparent friction poroelastic model (Cocco and Rice (2002), and references therein), which gives $\Delta p = B\Delta\sigma$, with B the Skepton coefficient. This yields

$$\Delta CFS = \Delta\tau - \mu'\Delta\sigma \tag{2}$$

where $\mu' = \mu(1 - B)$. The value of μ is a fixed input parameter.

2.2. Rate and state evolution of seismicity

Dieterich (1994) developed a formulation for the evolution of seismicity due to applied stresses, by considering the collective behavior of an infinite population of faults governed by rate-state dependent friction. The seismicity rate R evolves according to

$$R(t, x) = \frac{r_0(x)}{\gamma(t)\dot{\tau}_r} \tag{3}$$

with $\dot{\tau}_r$ the constant shear stressing rate and γ a constitutive parameter which evolves according to

$$d\gamma = \frac{1}{A\sigma}[dt - \gamma dS] \tag{4}$$

with $S = \tau - (\mu - \alpha)\sigma_{eff}$, where $\sigma_{eff} = \sigma - p$ and α is a positive non-dimensional constitutive parameter (Linker and Dieterich, 1992). For simplicity, CRS models assume $\alpha = \mu'$, so that S is the Coulomb stress as defined above. Alternatively, the equations can be formulated in terms of the parameters $A\sigma$ and the aftershock decay time, defined as $t_a \equiv A\sigma/\dot{\tau}_r$. $A\sigma$ and t_a are free parameters in CRS.

3. Model implementation

In this section, we outline the methods implemented within CRS and the program workflow. For detailed information on using the software, the reader is referred to the following sources: (1) the `readme` file; (2) the folder `doc`, containing for example input files; (3) the folder `tutorial`, containing a step-by-step example.

3.1. Spatial and temporal discretization

The space dependence of seismicity is represented by subdividing the model into user-defined grid points; we also provide an option to refine the grid spacing at run time, to perform more accurate Coulomb stress calculations. The temporal discretization employed to describe stressing history and to solve the equations is defined at run time, using non uniform step size to capture variations in stressing rate (see Section 2 of the Supplementary Material).

3.2. Input and output files

The input data provided by the user varies between applications, depending on modeling choices. Typically, the user provides the following information:

- A forecast template, i.e. a file with a list of grid points and magnitude bins;
- Start and end time for the forecast period;
- A list of stress sources (these can be in the form of a seismic catalog, slip models, focal mechanisms, or a combination of these);
- Rate-state parameters, or ranges to which the parameter search will be confined;
- Physical properties of the region, such as elastic moduli and background stress field.

In order to optimize rate-state parameters, a catalog must also be supplied, as well as a time range for the inversion period.

The main output file produced by CRS is a gridded forecast, binned in space and in magnitude and reporting the number of events in each bin during the forecast period. A time series of seismicity is also produced (seismicity rates and number of events summed over all grid points at a user-defined time interval). Other output files include a summary of the grid-search results (log-likelihood values for all combinations of rate-state parameters), the forecast from individual Monte Carlo iterations, and the model log-likelihood during the forecast period. A complete list of output files and their contents is given in the tutorial provided with the code.

3.3. Program workflow

Fig. 1 describes the work flow of the program. First, CRS sets up variables which remain constant during the entire execution of the program. These include structures describing the normalized background rate and the frequency-magnitude distribution, described in Sections 3.4 and 3.5 respectively. Typically, CRS performs two tasks: first, the optimal rate-state parameters (r_0, $A\sigma$, t_a) are found; then, the best parameters are used to produce a forecast. The forecast (small yellow box) in Fig. 1 essentially consists of the same procedure as the calculation of seismicity for the optimization period for a given set of rate-state parameters (larger yellow box); computationally, the main difference is that most output files are produced during the forecast. The main steps required to produce a forecast are: (1) calculating Coulomb stress changes and (2) calculating time-dependent seismicity from rate-state equations. In Sections 3.6 and 2.3 of the Supplementary Material, we describe each of these tasks. Aleatoric uncertainties are included by using a Monte Carlo method (Section 3.7). We explain the optimization procedure in Section 3.8. We finally outline the possibility to produce multiple forecasts based on alternative slip models.

3.4. Background seismicity rate $\tilde{r}_0(x, t)$

Here we are concerned with the normalized spatial distribution of background seismicity, $\tilde{r}_0(x, t) \equiv r_0(x, t)/\bar{r}_0$ and not with the average rate in the region. It is useful to distinguish between these quantities, since the latter can be treated as a free parameter (Section 3.8). Three options are available for describing background rate:

- Spatially uniform rate: while unrealistic, this modeling choice may be the most suitable for regions with low seismicity, where