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Case study

Computation of the gravity field and its gradient: Some applications

C.P. Dubey [a], V.M. Tiwari [a,b,*]

[a] CSIR-National Geophysical Research Institute, Hyderabad, India
[b] ESSO-National Centre for Earth Science Studies, Thiruvananthapuram, India

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ABSTRACT

New measuring instruments of Earth's gravity gradient tensors (GGT) have offered a fresh impetus to gravimetry and its application in subsurface exploration. Several efforts have been made to provide a thorough understanding of the complex properties of the gravity gradient tensor and its mathematical formulations to compute GGT. However, there is not much open source software available. Understanding of the tensor properties leads to important guidelines in the development of real three dimensional geological models. We present a MATLAB computational algorithm to calculate the gravity field and full gravity gradient tensor for an undulated surface followed by regular geometries like an infinite horizontal slab, a vertical sheet, a solid sphere, a vertical cylinder, a normal fault model and a rectangular lamina or conglomerations of such bodies and the results are compared with responses using professional software based on different computational schemes. Real subsurface geometries of complex geological structures of interest are approximated through arrangements of vertical rectangular laminas. The geological application of this algorithm is demonstrated over a horst-type structure of Oklahoma Aulacogen, USA and Vredefort Dome, South Africa, where measured GGT data are available.

1. Introduction

The Earth's gravity and gravity gradient anomalies provide important information for delineating geological structures of economic importance. The gravity gradient method is one of the geophysical tools used successfully to detect remote occurrences of target bodies and to define geological models with enhanced resolution. It is frequently employed in interpretation for isolating gravity anomalies (Murphy and Dickinson, 2009). The use of the gravity gradient in exploration is becoming more common in recent years due to the development of airborne gradiometry with an accuracy of ~2–5 eötvos unit (1E=0.1 mGal/km) over wavelengths of ~45 m (Dransfield and Christensen, 2013; Zuidweg and Mumaw, 2007) or ~100 km for the ongoing gradiometer satellite mission called GOCE (Herceg et al., 2014; Godah and Krynski, 2011), which can give a potential map easily over large, highly inaccessible undulating regions. Gravity gradiometers measure gradients of the gravity vector components in three Cartesian directions (Fig. 1) and measured components are used to produce the nine – component tensor, T_{ij}. Since the gravitational potential satisfies Laplace's equation, the trace of the symmetric tensor is equal to zero. Thus, there are only five independent elements (e.g., T_{xx}, T_{xy}, T_{xz}, T_{yy}, and T_{yz}) as $T_{ij}=T_{ji}$, where $i \neq j$. Furthermore, the T_{zz} component is often displayed as it closely relates to the subsurface geology (Pedersen and Rasmussen, 1990).

Conventional gravity data show the strength of the earth's gravity field but are less sensitive to the edges of bodies and contain no directional information. In contrast, gravity gradients directly recover sharp signal over the edges of structures and are closely related to the edges, corners, and center of mass of the causative bodies producing complex pattern of anomalies. For a simple positive density cube, a classic gravity map would show a diffused circular anomaly centered over the body. In contrast, the six gravity gradients provide a powerful tool for delineating the shape of the body (Saad, 2006). The vertical tensor component T_{zz} provides an estimate of maximum depth and predicts boundary information directly related to the geological body and the other components give close information related to the geometry of the body. The T_{xx} component effectively indicates the eastern and western edges of a feature, whereas the T_{yy} component indicates the northern and southern edges. The T_{xz} component divides the body into eastern and western halves approximately symmetrically and gives the central anomaly axis towards north–south direction; similarly T_{yz} component divides the body into northern and southern halves symmetrically and gives the central anomaly axis towards east–west direction. It also helps to show north–south and east–west trending edges. The T_{xy} component gives information about the four corners of near rectangular bodies and locates the center point of symmetrical bodies in case of alignment

* Corresponding author at: CSIR-National Geophysical Research Institute, Hyderabad, India.
E-mail addresses: virendra.m.tiwari@gmail.com, vmtiwari.ncess@gov.in (V.M. Tiwari).

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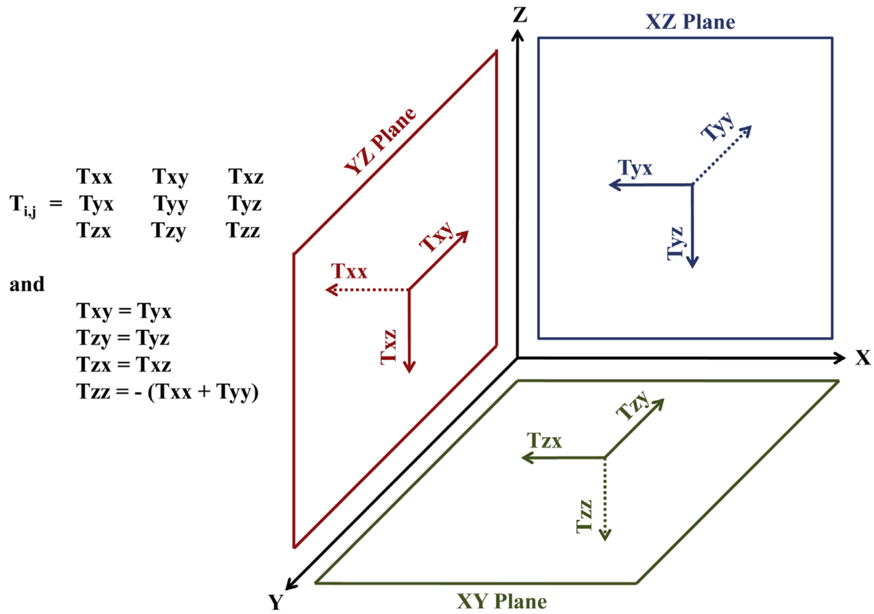

Fig. 1. Schematic diagram showing the gravity field vector G_x, G_y, G_z and full gravity gradient tensor components T_{xx}, T_{xy}, T_{xz}, T_{yy}, T_{yz} and T_{zz}.

of geological body with x and y directions.

2. Commutating gravity and its gradients for geological structures

In particular, gravity and gravity gradient data explore the subsurface geology originating from mass distribution in subsurface. Therefore, estimating the model parameters of causative sources such as location, depth, thickness, size, shape, extension, density variations etc., has a key importance in the interpretation stage. However the well-known complex nature of the full gravity gradient tensor (FTG) may quite complicate the interpretation procedure compare to the gravity alone (Saad, 2006). Therefore, for better understanding of the complex nature of FTG, it is thought to provide a detail discussion on computational algorithms of three dimensional regular shaped geometries and their behavior for the gravity field and their gradient components due to various geometrical shapes. Several researchers have proposed modeling approaches for computation of gravity and gravity gradient responses due to homogeneous polyhedral bodies (Okabe, 1979; Gotze and Lahmeyer, 1988; Barnet, 1976; Coggon, 1976; Pohánka, 1988; Yao and Changli, 2007). Bhattacharya (1964), Nagy (1966), and Plouff (1976) presented closed form mathematical equations for prism shaped bodies, whereas Talwani and Ewing (1960) and Talwani (1965) used numerical integration techniques for the computation of the fields due to models of arbitrary shape by dividing them into polygonal prisms or laminae. Some recent studies (e.g. Tsoulis, 2012) provided a mathematical formulation for computation of the full gravity gradient tensor from a polyhedral source. The present study utilizes theory of gravity and gravity gradient effects of a rectangular prism or rectangular lamina (Talwani, 2011) and presents MATLAB algorithms for the computation of the primary gravity field and their derivatives to each coordinate direction for regular shaped geometries like the rectangular prism, dipping fault, spherical body, vertical cylinder body and two dimensional geometries like the semi-infinite horizontal slab, and dike, all of uniform density.

2.1. Computations for regular geometries

Many geological features are approximated by 2D models like an infinite dike or a geological contact for computational simplifications. The interpretation of 2D and 2.5D potential field models is simple but might be far from reality since the real geological bodies are mostly three dimensional structures. Following sections discusses both two dimensional and three dimensional geometries with an emphasis on the three dimensional regular shaped geometries used in analyzing asymmetrical three dimensional arrangements in the subsurface. Each of the geometrical bodies used is of uniform density, although in many geological situations, the density of a particular structure may vary. This is particularly true in sedimentary rocks where the density increases with depth as a result of compaction. Often, this may be allowed using simple functional forms to approximate the density variation with depth as well as its lateral variations. However, for the purpose of this paper, the model assumes constant density for each element of a model and variations in density are simulated by the use of separate elements.

2.2. Geometries: semi-infinite horizontal slab and vertical sheet/dike

For many exploration purposes, it is common to assume that the body producing a gravity anomaly is two dimensional in nature. In particular, the removal of one spatial dimension allows greater complexity to be built into the remaining two dimensions of the model as well as making the computation of the simulation faster. Also mineralized zones are often found over linear features like shear zones, faults and so on that can be approximated as two directional features. The gravity gradients in the two-dimensional case are quite simple, since the tensor has only four components with the off-diagonal components equal by symmetry of the tensor and the diagonal elements equal in magnitude but opposite in sign (Dransfield, 1994). The potential at a point much closer to the center of the elongated body than its end is independent of the distance to the ends and therefore the components of gravity and gravity gradients in that direction are zero.

Here, we present the gravity and horizontal gradient response for two simple geometries: the semi-infinite horizontal sheet and the vertical sheet (Fig. 2). Their analytical expressions are