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Fair implementation of diversity in school choice ☆

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ABSTRACT

Many school districts have objectives regarding how students of different races, ethnicity or religious backgrounds should be distributed across schools. A growing literature in mechanism design is introducing school choice mechanisms that attempt to satisfy those requirements. We show that mechanisms based on the student-proposing deferred acceptance may fail to satisfy those objectives, but that by using instead the school-proposing deferred acceptance together with a choice function used by the schools, which incorporates a preference for satisfying them, can optimally approximate the diversity objectives while still satisfying an appropriate fairness criterion. We provide analytical results which show that the proposed mechanism has a general ability to satisfy those objectives, as opposed to some currently proposed mechanisms, which may yield segregated assignments.

1. Introduction

Over the last five decades a multitude of policies have been implemented, with varying degrees of success, to reduce historical and emerging racial, religious, and ethnic segregation at the school level. Most of the policies used to achieve that objective aim to either establish maximum quotas for the so-called majority students or to give higher priority to minority students in either all or part of the seats available.

Since the seminal work on the subject by Abdulkadiroğlu and Sönmez (2003), a growing number of papers have used mechanism design principles to obtain school assignments that achieve some balance between diversity objectives, fairness, efficiency, and other properties. One class of such mechanisms, which we denote *affirmative action mechanisms*, expands the set of schools that certain types of students have access to by giving them higher priority and/or reserving some seats in the schools to be filled by those students, making the seats otherwise available to everyone.[1] One example of an affirmative action mechanism is giving higher priority to racial minorities for a number of seats in schools (Hafalir et al., 2013; Ehlers et al., 2014; Aygün and Bó, 2013). Another class of mechanisms takes diversity as an objective instead, and accommodates

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[1] The term affirmative action is normally used in a broader sense across the literature. Mechanisms such as those in Abdulkadiroğlu and Sönmez (2003) and Abdulkadiroğlu (2005) are denoted in those papers as implementing *affirmative action* while in our terminology those are *diversity implementation* mechanisms.

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other properties, such as fairness or constrained efficiency. We denote that class of mechanisms *diversity implementation mechanisms*. Mechanisms with majority quotas (which enforce a maximum number of "majority type" students in each school) or others that enforce certain ratios among types of students are examples of diversity implementation mechanisms.

For problems such as university admission – which is in many cases determined by student performance in tests and high-school grades – affirmative action mechanisms could increase the diversity of cohorts by improving the access of minority students to more competitive universities. In the case of school choice, however, that it is not necessarily the case. Typically, the criteria for admission rely on aspects such as residence location, presence of siblings in the school, special needs, etc. That is, minority students are not necessarily disadvantaged with respect to others in their access to desired schools, and thus the use of such mechanisms may not help in obtaining more diverse groups of students.

We introduce a new diversity implementation mechanism that differs from others available in the literature in two main aspects: the incorporation of diversity objectives as an element of fairness and a more pragmatic interpretation of those objectives, where a given distribution of types in a school is used as a desired target instead of a strict objective.

From a theoretical perspective, one key aspect of this paper is our use of the school-proposing deferred acceptance procedure while using a choice function for the schools that incorporates a preference for groups of students that satisfy the diversity requirements. While college admissions problems are two-sided matching problems in which the welfare and incentives of both sides are under consideration, in a school choice problem the seats in the schools are simply objects to be allocated to students. Therefore the school's choice function can be designed in such a way that the property of stability and the school-optimality of the stable allocations selected *induces the desired properties on the allocation*. Moreover, as shown in section 3, the change from using the student-proposing to school-proposing deferred acceptance has significant effects on the satisfaction of diversity objectives.

1.1. Relation with the literature

The Student-Proposing Deferred Acceptance and the College-Proposing Deferred Acceptance mechanisms (SPDA and CPDA) were first introduced by Gale and Shapley (1962). While Dubins and Freedman (1981) show that when using the SPDA as a direct mechanism no student or group of students can be made better-off by misrepresenting their preferences, Gale and Sotomayor (1985) show that this is not normally the case when using CPDA. Furthermore, Roth (1985) shows that there is no stable mechanism that is immune to manipulation by colleges.[2]

The incentive and welfare properties of both mechanisms come into play in the context of college admissions in Balinski and Sönmez (1999). In their model there is no need for strategic or welfare considerations on the part of colleges. As a result, the SPDA is suggested as the ideal mechanism for the student placement problem.

The subsequent literature on college admissions and school choice, as well as their applications, focuses on the use of the SPDA procedure (see Abdulkadiroğlu and Sönmez, 2003; Abdulkadiroğlu et al., 2005, 2006, 2009). When concerns about the diversity of the distribution of students across schools were introduced in the mechanisms, that choice persisted (Ehlers et al., 2014; Echenique and Yenmez, 2015; Erdil and Kumano, 2012; Kominers and Sönmez, 2012; Hafalir et al., 2013). As shown in section 3, however, those mechanisms may to a great extent fail to affect the distribution of students by type in the schools. By combining the use of the CPDA procedure with a choice function used by the schools which represents a preference for satisfying the diversity objectives, we are able to obtain assignments that implement (or approximate) those objectives in a wider range of scenarios while still satisfying a fairness criterion.

The paper proceeds as follows. Section 2 introduces the model and the SPDiv mechanism. Section 3 presents the analytical results of the outcomes generated by the SPDiv mechanism and student-proposing affirmative action mechanisms. Proofs omitted from the main text can be found in the Appendix.

2. Model

A **school choice with diversity problem** consists of a tuple $\left\langle S, C, T, \tau, q, \underline{q}, \succ_S, \succ_C \right\rangle$:

1. A finite set of **students** $S = \{s_1, \ldots, s_n\}$
2. A finite set of **schools** $C = \{c_1, \ldots, c_m\}$
3. A finite set of **types** $T = \{t_1, \ldots, t_k\}$
4. A function $\tau : S \to T$ where $\tau(s)$ is the type of student s. We denote by $S^t(I)$ the set of students in $I \subseteq S$ of type t, that is, $S^t(I) = \{s \in I : t = \tau(s)\}$.
5. A capacity vector $q = \left(q_{c_1}, \ldots, q_{c_m}\right)$ where q_c is the **capacity** of school $c \in C$.
6. For each school c, a vector $q_c^T = \left(q_c^{t_1}, \ldots, q_c^{t_k}\right)$ of **diversity objectives**, where q_c^t is the minimum desired number of students with type t at school c, where $\sum_{t \in T} q_c^t \leq q_c$. Let $\underline{q} = \left(q_{c_1}^T, \ldots, q_{c_m}^T\right)$.

[2] Whereas Dubins and Freedman (1981) assume that the colleges' selection of the students can be represented by ranking them and choosing the most preferred ones up to a capacity constraint, similar results are shown for more general choice functions in, among others, Hatfield and Kojima (2010) and Abdulkadiroğlu (2005).