Games and Economic Behavior 81 (2013) 69–85

Contents lists available at SciVerse ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Beyond fictitious play beliefs: Incorporating pattern recognition and similarity matching

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ARTICLE INFO

Article history:
Received 2 July 2012
Available online 9 May 2013

JEL classification:
C70
C72
C73
C91
C52
C53

Keywords:
Learning
Pattern recognition
Beliefs
Repeated games
Memory
Cognitive models
Behavioral game theory
ACT-R

ABSTRACT

Belief models capable of detecting 2- to 5-period patterns in repeated games by matching the current historical context to similar realizations of past play are presented. The models are implemented in a cognitive framework, ACT-R, and vary in how they implement similarity-based categorization—using either an exemplar or a prototype approach. Empirical estimation is performed on the elicited-belief data from two experiments (Nyarko and Schotter, 2002; Rutström and Wilcox, 2009) using repeated games with a unique, albeit significantly different, stage mixed-strategy Nash equilibrium. Model comparisons are performed by cross-validation both within and between these two datasets, and using data from completely unrelated non-strategic tasks. Subjects' beliefs are best described by 2-period pattern detection. Parameter estimates exhibited considerable instability across the two belief-elicitation datasets, and surprisingly, using median values from a wide variety of unrelated studies led to better predictions.

1. Introduction

A significant portion of the experimental game theory literature is devoted to games with a unique mixed-strategy Nash equilibrium (MSNE), which prescribes the empirical marginal distribution over actions to be played at equilibrium. Furthermore, in repeated games play must be independently distributed over rounds; players have no vested interest in searching for historical patterns if an opponent is randomizing effectively. Empirical investigations using both non-strategic and strategic tasks cast significant doubt on the validity of this assumption: people are generally not effective at randomizing[1] and

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[1] Studies in non-strategic tasks find a deficit in the ability of humans to randomize (Bar-Hillel and Wagenaar, 1991; Rapoport and Budescu, 1997). In strategic games, results are mixed but the weight of evidence points to a predominance of inefficient randomization when subjects are not experts in the task that they face.

Professionals in their field of specialization/experience are efficient randomizers in soccer penalty kicks (Palacios-Huerta, 2003; Chiappori et al., 2002) but not in tennis serves (Walker and Wooders, 2001)—although even in the latter the empirical marginal probabilities are in line with the MSNE prescription. Professionals' ability to randomize efficiently in their domain of specialization is found not to transfer to other settings in the lab (Chiappori et al., 2002)—professionals did not randomize well even when playing against computerized Nash opponents. Although Palacios-Huerta and Volij (2008) find that professionals in the lab randomize more efficiently than non-professional subjects and generally conform more closely to the MSNE, Wooders (2010)

0899-8256/$ – see front matter
http://dx.doi.org/10.1016/j.geb.2013.04.005

actively search for, and recognize, patterns in their environment.[2] The two most relevant papers incorporating pattern recognition directly into beliefs are Spiliopoulos (2012) and Rutström and Wilcox (2009). I will advance this literature by proposing variants of pattern-detection models based on cognitive processes, selecting amongst these variants using novel model evaluation methods, and investigating parameter stability and predictive performance across experimental tasks.

Research on pattern recognition in strategic games is surprisingly scarce. Theoretical convergence results are difficult to attain and require relatively strict assumptions about the belief updating process (Aoyagi, 1996; Sonsino, 1997; Fudenberg and Levine, 1998). Only a handful of empirical papers have examined pattern detection in games. In the context of a repeated Battle-of-the-Sexes game, Sonsino and Sirota (2003) empirically investigate the resulting patterns of alternating equilibrium outcomes. West and Lebiere (2001) examine agents modeled as simple neural networks that condition their action choices on the recent history of play. Lebiere and West (1999) and Lebiere et al. (2000) examine the behavior of agents modeled on the ACT-R framework (Anderson, 2007) and find qualitative similarities with summary statistics from experimental data such as aggregate payoff performance and empirical marginal probabilities of action choices. Spiliopoulos (2013) presents evidence of 2-period pattern detection influencing marginal and conditional distributions over action choices, thereby explaining the strategic adaptation of human subjects to different computer opponents in a repeated 2×2 game with a unique MSNE.

The following two studies specifically examine pattern recognition in belief formation. Spiliopoulos (2012) investigates a repeated constant-sum 2×2 game with fixed partner matching using belief elicitation, i.e. subjects reported the probability they assigned to an opponent's action. Subjects were modeled as being one of two types. The first (non-pattern detecting) type formed beliefs according to standard weighted fictitious play (Cheung and Friedman, 1997) and the second type according to a 2-period pattern detecting variant of weighted fictitious play. A large proportion of subjects were identified as the second type, and the probability of a subject belonging to the second type increased with the degree of exploitability—in terms of deviation from independent behavior across rounds—that an opponent exhibited. Importantly, evidence of pattern recognition was found both in the elicited beliefs and the realized action choices of subjects. Rutström and Wilcox (2009) examine the behavioral effects of two belief-elicitation procedures in a repeated 2×2 game with fixed matching. Although not specifically investigating pattern recognition, they modeled belief formation as a convex combination of standard weighted fictitious play and a 2-period weighted fictitious play model that conditioned on both players' history of play—they estimated the weight attributed to the pattern-detecting model to lie between 0.214 and 0.352, depending on the belief-elicitation treatment.

This paper advances the literature in the following directions. Firstly, instead of arbitrarily assuming an upper bound of 2-period pattern recognition, the depth employed by subjects will be empirically determined from a permissible range of 2- to 5-period patterns.[3]

Secondly, the process for matching historical patterns with the current history will allow for two different types of similarity-based matching (exemplar and prototype)—similar, but not necessarily identical, histories will also influence the belief formation process. The exemplar approach to categorization, pioneered by Medin and Schaffer (1978), Nosofsky (1984, 1986) and extended by Nosofsky and Palmeri (1997), performs similarity comparisons individually with respect to each observation. In contrast, the prototype approach advocated by Posner and Keele (1968) and Reed (1972) computes similarity comparisons with prototypes of each category that are created by the averaging of observations. I directly compare the exemplar and prototype models by embedding them in the ACT-R (Anderson, 1996, 2007; Anderson and Lebiere, 1998) cognitive framework that has performed successfully in a wide range of domains of human cognition.[4] Close attention will be paid to realistic cognitive constraints and how they relate to the belief formation process. Specifically, I will examine the implications of working memory constraints of 4 ± 1 items (Cowan, 2001),[5] as well as the constraints on the long-term declarative-memory module with respect to forgetting, noisy retrieval, and context effects.

Finally, model selection is performed by examining the generalizability of models across two different datasets/experiments, the elicited-belief experiments of Nyarko and Schotter (2002), or NS for short, and Rutström and Wilcox (2009),

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finds that other implications of the MSNE are not consistent with their conclusion. Further research seems needed to clarify the conditions under which professionals transfer specialized knowledge to other domains.

The evidence regarding inexperienced subjects is more homogeneous in finding inefficient randomization, starting from the reexamination of O'Neill (1987) in Brown and Rosenthal (1990), and the aforementioned studies pitting professional versus inexperienced subjects (Palacios-Huerta and Volij, 2008; Levitt et al., 2010), although Wooders (2010) rejects the minimax hypothesis only for one of four actions in inexperienced subjects. Notably, Shachat (2002) does not find evidence of subjects playing the MSNE even when an external randomization device is made available. Finally, Scroggin (2007) finds significant evidence of exploitable patterns that can be explained by a local representativeness bias in two classic experiments (Mookherjee and Sopher, 1994; Rapoport and Budescu, 1992).

[2] In non-strategic tasks, the sequence-learning literature concludes that humans are capable of detecting patterns of up to 5 temporal lags (Clegg et al., 1998; Gomez, 1997; Nissen and Bullemer, 1987; Remillard and Clark, 2001). Furthermore, searching for patterns appears to be a widespread human trait, as subjects search and believe they have found patterns even in random sequences (Wolford et al., 2004; Gaissmaier and Schooler, 2008).

[3] This is important, as not accounting for pattern recognition can lead to biased parameter estimates due to model misspecification (Spiliopoulos, 2012).

[4] Examples include problem solving (Gunzelmann and Anderson, 2001), decision making (Gonzalez et al., 2003; Gonzalez and Dutt, 2011; Marewski and Melhorn, 2011), visual search and attention (Anderson et al., 1997), cognitive arithmetic (Lebiere, 1999), spatial reasoning (Boeddinghaus et al., 2006) and language processing (Budiu and Anderson, 2004)—the reader is referred to http://act-r.psy.cmu.edu/publications/index.php for an extensive list of ACT-R related papers.

[5] The classic paper on working memory span by Miller (1956) concludes that the constraint is 7 ± 2 items. However, Cowan (2001) surveys the recent literature and argues strongly in favor of revising this number to 4 ± 1 items.