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Interplay of subexponential and dependent insurance and financial risks

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HIGHLIGHTS

- We study the ruin probability of an insurer who makes risky investments.
- A discrete-time risk model is employed to accommodate the interplay of the insurance and financial risks.
- The insurance and financial risks are dependent via a conditional copula density.
- For the subexponential case, an asymptotic formula for the finite-time ruin probability is established.
- The proof makes use of the subexponentiality of the product of two dependent random variables.

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ABSTRACT

We are interested in the ruin probability of an insurer who makes risky investments and hence faces both insurance and financial risks. Assume that the insurance and financial risks over individual periods, (X_i, Y_i), $i \in \mathbb{N}$, form a sequence of independent and identically distributed copies of a generic pair (X, Y) and that the pair (X, Y) possesses a weak dependence structure described via its copula. For the subexponential case, we obtain an asymptotic formula for the finite-time ruin probability as our main result, which extends a few recent works on the topic.

1. Introduction

Consider an insurer who makes risky investments and hence faces both insurance and financial risks. We use a discrete-time risk model to accommodate the two risks. Within each period i, the insurance risk is quantified as the net insurance loss variable X_i equal to claims plus expenses minus premiums over the period, and the financial risk is quantified as the stochastic present value factor Y_i equal to the reciprocal of the stochastic accumulation factor calculated according to overall returns on investments over the same period. Thus, the sum

$$S_n = \sum_{i=1}^{n} X_i \prod_{j=1}^{i} Y_j,\ n \in \mathbb{N}, \tag{1.1}$$

represents the stochastic present value of aggregate net insurance losses up to time n. As usual, the insurer who holds an initial capital $x \geq 0$ is regarded as ruined if this stochastic present value process $\{S_n, n \in \mathbb{N}\}$ upcrosses x. Precisely, the probability of ruin by time n is defined to be

$$\psi(x; n) = P\left(\max_{1\leq m\leq n} \sum_{i=1}^{m} X_i \prod_{j=1}^{i} Y_j > x\right),\ n \in \mathbb{N}. \tag{1.2}$$

As investments become a more and more significant component of any insurance business nowadays, the insurance and financial risks described above are two fundamental risks which every insurer is exposed to and should be carefully addressed in conducting solvency assessment of an insurance business. This discrete-time risk model serves as an effective platform for the interplay of the two risks. Initiated by Nyrhinen (1999, 2001) and Tang and Tsitsiashvili (2003, 2004), the study of the ruin

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probability (1.2) in the presence of both insurance and financial risks has attracted enormous attention from researchers. Paulsen (2008) and Asmussen and Albrecher (2010) made good reviews of some early works on this study. Recent works include Nyrhinen (2010, 2016), Chen (2011), Yang and Konstantinides (2015), Li and Tang (2015), Lehtomaa (2015), Tang and Yuan (2015, 2016), Chen et al. (2015), Yang and Yuen (2016), and Chen and Yuan (2017).

In the majority of works on this study, it is assumed that (X_i, Y_i), $i \in \mathbb{N}$, form a sequence of independent and identically distributed (i.i.d.) copies of a generic pair (X, Y). Denote by F on $\mathbb{R}$ and G on $\mathbb{R}_+$ the two marginal distribution functions of (X, Y), and by $C(\cdot,\cdot)$ its copula, so that

$$P(X \le x, Y \le y) = C(F(x), G(y)), \qquad (x, y) \in \mathbb{R}^2. \tag{1.3}$$

In particular, Chen (2011) and Chen et al. (2015) studied the ruin probability (1.2) under the assumptions that F is a subexponential distribution and that C is a bivariate Farlie–Gumbel–Morgenstern (FGM) copula of the form

$$C(u, v) = uv\,(1 + \theta(1-u)(1-v)), \qquad (u, v) \in [0, 1]^2, \tag{1.4}$$

where the parameter $\theta \in [-1, 1]$ governs the strength of dependence. Later on, Yang and Konstantinides (2015) extended the works to more general copulas but under more restrictive assumptions on the marginal distribution function F.

In this paper, assuming the subexponentiality of F and a weak dependence on (X, Y) described via the copula C (see Assumption 2.1), we obtain an asymptotic formula for the ruin probability (1.2). This result unifies and extends a few recent works on the topic including Chen (2011), Chen et al. (2015), Yang and Konstantinides (2015), and Tang and Yuan (2016).

The rest of this paper consists of two sections. In Section 2, after making some notational conventions and collecting necessary preliminaries on subexponential distributions, we introduce and explain an assumption on the dependence between the two risks and we show the main result of the paper. In Section 3, after preparing a number of lemmas we complete the proof of the main result.

2. The main result

2.1. Notational conventions

Throughout the paper, all limit relationships are according to $x \to \infty$ unless otherwise stated. For two positive functions $h_1(\cdot)$ and $h_2(\cdot)$, we write $h_1(x) \lesssim h_2(x)$ or $h_2(x) \gtrsim h_1(x)$ if $\limsup h_1(x)/h_2(x) \le 1$ and write $h_1(x) \sim h_2(x)$ if $\lim h_1(x)/h_2(x) = 1$. We also write $h_1(x) \asymp h_2(x)$ if $0 < \liminf h_1(x)/h_2(x) \le \limsup h_1(x)/h_2(x) < \infty$. For simplicity, we say that a measurable function $a(\cdot)$ on $\mathbb{R}_+$ is an auxiliary function if it satisfies

- $0 \le a(x) < x/2$,
- $a(x) \uparrow \infty$, and
- $a(x)/x \downarrow 0$.

For two distribution functions F and G, write $F * G$ their sum convolution and write $F \otimes G$ their product convolution. In other words, introducing two independent random variables X and Y distributed by F and G, respectively, $F * G$ denotes the distribution function of the sum $X+Y$ while $F \otimes G$ denotes the distribution function of the product XY. Higher fold convolutions can be introduced similarly, but we let the notation speak for itself.

2.2. Subexponential distributions

A distribution function F on $\mathbb{R}_+ = [0, \infty)$ is said to be subexponential, written as $F \in \mathcal{S}$, if F has an ultimate right tail in the sense that $\overline{F}(x) = 1 - F(x) > 0$ for all $x \in \mathbb{R}_+$ and if

$$\overline{F * F}(x) \sim 2\overline{F}(x).$$

More generally, a distribution function F on $\mathbb{R}$ is still said to be subexponential if the distribution function $F_+(x) = F(x)1_{(x \ge 0)}$ is subexponential.

It is well known that every subexponential distribution function F is long tailed, written as $F \in \mathcal{L}$, in the sense that the relation

$$\overline{F}(x + y) \sim \overline{F}(x)$$

holds for some (or, equivalently, for all) $y \ne 0$; see Lemma 1.3.5(a) of Embrechts et al. (1997). By the definition, it is easy to see that $F \in \mathcal{L}$ if and only if there is an auxiliary function $a(\cdot)$ such that the relation

$$\overline{F}(x + ca(x)) \sim \overline{F}(x)$$

holds for every $c \in \mathbb{R}$; see also Lemma 4.1 of Li et al. (2010) for a proof of this assertion.

The interested reader is referred to the monographs Bingham et al. (1987), Embrechts et al. (1997), Asmussen and Albrecher (2010), and Foss et al. (2011) for comprehensive treatments on the subexponential and related heavy-tailed distribution classes.

2.3. On the dependence structure

As mentioned before, in this paper we assume that (X_i, Y_i), $i \in \mathbb{N}$, form a sequence of i.i.d. copies of a generic random pair (X, Y) with marginal distribution functions F on $\mathbb{R}$ and G on $\mathbb{R}_+$. Let (X, Y) possess a copula $C(u, v)$ for $(u, v) \in [0, 1]^2$. We make the following assumption on the dependence structure of (X, Y) via its copula C:

Assumption 2.1. Let (U, V) be a uniform vector distributed by the copula C of the generic random pair (X, Y). There is a nonnegative function $\gamma(v)$ on $(0, 1)$, bounded away from both 0 and ∞ in a left neighborhood of $v = 1$, such that the following two uniform convergences hold:

$$\lim_{u \uparrow 1} \sup_{v \in (0,1)} \left| \frac{d}{dv} P(V \le v \,|U > u) - \gamma(v) \right| = 0 \tag{2.1}$$

and

$$\limsup_{u \downarrow 0} \sup_{v \in (0,1)} \frac{P(V > v \,|U \le u)}{1 - v} < \infty. \tag{2.2}$$

Remark 2.1. We make some remarks on Assumption 2.1.

(i) For notational convenience, we shall simply rewrite the uniform convergence (2.1) as

$$\lim_{u \uparrow 1} P(V \in dv \,|U > u) = \gamma(v) dv,$$

but we point out that this should be properly understood in terms of an integral with respect to dv over a relevant region. For example, for a measurable function $q(\cdot)$ bounded over $[0, 1]$, by the uniform convergence above, we can safely derive

$$\lim_{u \uparrow 1} E[q(V) \,|U > u] = \lim_{u \uparrow 1} \int_0^1 q(v) P(V \in dv \,|U > u) = \int_0^1 q(v)\gamma(v) dv.$$

(ii) It is straightforward to verify that the FGM copula (1.4) fulfills Assumption 2.1 with

$$\gamma(v) = 1 - \theta + 2\theta v, \qquad v \in [0, 1].$$