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Some comparison results for finite-time ruin probabilities in the classical risk model

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ABSTRACT

This paper aims at showing how an ordering on claim amounts can influence finite-time ruin probabilities. Until now such a question was examined essentially for ultimate ruin probabilities. Over a finite horizon, a general approach does not seem possible but the study is conducted under different sets of conditions. This primarily covers the cases where the initial reserve is null or large.

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1. Introduction

The evaluation of ruin probabilities strongly depends on the distribution of the claim amounts. Given two claim distributions, it is natural to ask which one implies larger values of ruin probabilities. This topic has been investigated for a long time (see e.g. the book of Goovaerts et al., 1990). Most often, the attention is focused on ruin over an infinite time horizon. Intuitively, one expects that a more variable claim amount increases the ultimate ruin probability. Such a result was first proved by Michel (1987) for the classical risk model using the convex order of claim sizes. Other stochastic orderings have been considered to tackle different situations or model assumptions. A nice paper by Klüppelberg (1993) uses asymptotic orders to compare ruin probabilities when initial reserves are large, for light- and heavy-tailed claim distributions.

To the best of our knowledge, the influence of claim sizes on finite-time ruin probabilities has been studied very little so far. A notable exception is the paper by De Vylder and Goovaerts (1984) who obtained some interesting results for the compound Poisson risk model. In particular, they showed that contrary to the infinite time case, a more dangerous claim amount in the convex order sense does not necessarily imply larger ruin probabilities over finite-time horizons.

The present paper deals also with the classical risk model. Claims occur according to a Poisson process $\{N_t,\ t \geq 0\}$ of rate $\lambda > 0$ and claim amounts $\{X_i,\ i \geq 1\}$ that are independent and identically distributed (i.i.d.) positive random variables (distributed as X) with distribution function F and mean μ. So, the aggregate claim amount up to time t is $S_t = \sum_{i=1}^{N_t} X_i$. The company has an initial reserve of level $u \geq 0$ and receives premium at a constant rate c. The probability of ruin (resp. non-ruin) before time $t \geq 0$ is denoted by $\psi(u, t)$ (resp. $\phi(u, t)$). A positive safety loading factor η is defined by writing $c = \lambda\mu(1 + \eta)$. It guarantees that the ruin probability over an infinite time horizon, $\psi(u) = 1 - \phi(u)$, is less than 1 and tends to 0 as $u \to \infty$. For an overview of ruin theory, see e.g. the books of Dickson (2016) and Asmussen and Albrecher (2010).

Our aim here is to go further in the analysis of the possible influence of the claim amounts on the finite-time ruin probabilities. A simple unifying approach does not seem possible and the problem will be examined under several sets of conditions. As a mathematical tool, we will use different well-known stochastic orderings. Much of the theory on stochastic orders can be found e.g. in the books of Belzunce et al. (2015), Denuit et al. (2006), Müller and Stoyan (2002) and Shaked and Shanthikumar (2007). Furthermore, we will also use several asymptotic orders, less standard, that were introduced by Klüppelberg (1993).

The paper is organized as follows. In Section 2, we bring some complements to the analysis made by De Vylder and Goovaerts (1984). These concern the special cases where the initial reserve is null. In Section 3, we obtain a comparison result for the stop-loss transform of ruin probabilities. Such a result gives a partial perspective on the comparison of the probabilities themselves. In Section 4, we establish an asymptotic comparison of ruin probabilities as the initial reserve is large. Our study is directly inspired from the approach of Klüppelberg (1993) for the case of an infinite time horizon. In Section 5, we derive a comparison result for the

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time dependent Lundberg coefficient. This enables us to discuss the situation where the initial reserve and the time horizon are both large.

2. Null initial reserve

We begin by recalling the definitions of some stochastic orders that will be useful in the paper. The reader is referred e.g. to Shaked and Shanthikumar (2007), denoted as S–S in the following.

Let $X^{(1)}$ and $X^{(2)}$ be two non-negative random variables with distribution functions $F_1 = 1 - \overline{F}_1$ and $F_2 = 1 - \overline{F}_2$ and finite means μ_1 and μ_2, respectively. One says that $X^{(1)}$ precedes $X^{(2)}$ in the usual stochastic order, denoted as $X^{(1)} \preceq_{st} X^{(2)}$, if

$$\overline{F}_1(x) \le \overline{F}_2(x) \quad \text{for all } x \ge 0.$$

The latter is also equivalent to the inequality $E[g(X^{(1)})] \le E[g(X^{(2)})]$ for any non-decreasing function g such that the expectations exist.

The stochastic order compares the sizes of the risks. On the other hand, the convex order focuses on their variabilities and allows us to compare two risks with identical means. One says that $X^{(1)}$ precedes $X^{(2)}$ in the convex order, denoted as $X^{(1)} \preceq_{cx} X^{(2)}$, when $\mu_1 = \mu_2$ and

$$\int_x^{\infty} \overline{F}_1(u)\, du \le \int_x^{\infty} \overline{F}_2(u) du \quad \text{for all } x \ge 0. \tag{2.1}$$

The latter is also equivalent to the inequality

$$E[(X^{(1)} - x)_+] \le E[(X^{(2)} - x)_+] \quad \text{for all } x \ge 0,$$

where, for any real r, r_+ denotes the positive part of r (i.e. $r_+ = r$ if $r \ge 0$ and $r_+ = 0$ if $r < 0$). Equivalently, $X^{(1)} \preceq_{cx} X^{(2)}$ if and only if $\mu_1 = \mu_2$ and $E[h(X^{(1)})] \le E[h(X^{(2)})]$ for all convex functions $h : \mathbb{R}^+ \to \mathbb{R}$, provided the expectations exist.

Only random variables with the same means can be compared by the convex order. The Lorenz order and the increasing convex order combine the aspects of size (as $\preceq_{st}$) and variability (as $\preceq_{cx}$). One says that $X^{(1)}$ is smaller than $X^{(2)}$ in the Lorenz order, denoted as $X^{(1)} \preceq_{Lorenz} X^{(2)}$, when

$$\frac{X^{(1)}}{\mu_1} \preceq_{cx} \frac{X^{(2)}}{\mu_2}.$$

$X^{(1)}$ is said to be smaller than $X^{(2)}$ in the increasing convex order, denoted as $X^{(1)} \preceq_{icx} X^{(2)}$, when (2.1) holds true. Equivalently, $X^{(1)} \preceq_{icx} X^{(2)}$ if and only if $E[h(X^{(1)})] \le E[h(X^{(2)})]$ for all non-decreasing convex functions $h : \mathbb{R}^+ \to \mathbb{R}$, provided the expectations exist. Obviously, when $\mu_1 = \mu_2$, the increasing convex order is equivalent to the convex order. The increasing convex order is also named the stop-loss order as $E[(X - x)_+]$ is the expected reinsurance payment under a stop-loss reinsurance treaty with retention x.

Similarly, an increasing concave order, denoted as $X^{(1)} \preceq_{icv} X^{(2)}$, is defined by requiring $E[h(X^{(1)})] \le E[h(X^{(2)})]$ for all non-decreasing concave functions $h : \mathbb{R}^+ \to \mathbb{R}$, provided the expectations exist. This condition is equivalent to

$$E[(X^{(1)} - x)_-] \ge E[(X^{(2)} - x)_-] \quad \text{for all } x \ge 0,$$

where, for any real r, r_- denotes the negative part of r (i.e. $r_- = -r$ if $r \le 0$ and $r_- = 0$ if $r > 0$).

Now, let us compare the finite-time ruin probability for risk models with no initial reserves ($u = 0$). An index j or a superscript (j), $j = 1, 2$, will be added in the notation to distinguish the models. When $u = 0$, it is well-known that the non-ruin probability before time t is simply given by

$$\phi(0, t) = \frac{1}{ct} E\left[(ct - S_t)_+\right] \quad (\text{i.e. also } \frac{1}{ct} E\left[(S_t - ct)_-\right]). \tag{2.2}$$

This last equation is often referred as the Takàcs formula (see Takács, 1967). From this result, we can establish the following proposition.

Proposition 1. *If $\lambda_1 \le \lambda_2$ and $X^{(1)}/c_1 \preceq_{icv} X^{(2)}/c_2$, then $\psi_1(0, t) \le \psi_2(0, t)$ for all $t > 0$.*

Proof. Since $\lambda_1 \le \lambda_2$, we have $N_t^{(1)} \preceq_{st} N_t^{(2)}$ and hence $N_t^{(1)} \preceq_{icv} N_t^{(2)}$ (see Theorem 4.A.34 in S–S). Furthermore, since $X^{(1)}/c_1 \preceq_{icv} X^{(2)}/c_2$, we get $S_t^{(1)}/c_1 \preceq_{icv} S_t^{(2)}/c_2$ (see Theorem 4.A.9 in S–S). So, by definition of the increasing concave order, we obtain

$$E[(S_t^{(1)}/c_1 - t)_-] \ge E[(S_t^{(2)}/c_2 - t)_-],$$

and hence $\psi_1(0, t) \le \psi_2(0, t)$ by virtue of (2.2). □

We note that the condition $X^{(1)}/c_1 \preceq_{icv} X^{(2)}/c_2$ implies $\mu_1/c_1 \le \mu_2/c_2$. In particular, if $\eta_1 = \eta_2$, then $\lambda_1 \ge \lambda_2$. In the case where $\mu_1/c_1 = \mu_2/c_2$, $X^{(1)}/c_1 \preceq_{icv} X^{(2)}/c_2$ is equivalent to $X^{(2)}/c_2 \preceq_{cx} X^{(1)}/c_1$ (see Theorem 4.A.35 in S–S).

Proposition 1 is a slight generalization of Theorem 4 in De Vylder and Goovaerts (1984) which states that $\psi_1(0, t) \le \psi_2(0, t)$ if $c_1 = c_2$, $\lambda_1 = \lambda_2$ and $X^{(2)} \preceq_{cx} X^{(1)}$. Indeed, these conditions directly imply $X^{(1)}/c_1 \preceq_{icv} X^{(2)}/c_2$.

Corollary 1. *If $\lambda_1 = \lambda_2$, $\eta_1 = \eta_2$ and $X^{(1)} \preceq_{Lorenz} X^{(2)}$, then $\psi_1(0, t) \ge \psi_2(0, t)$ for all $t > 0$. In addition, if $\mu_1 = \mu_2$ holds too, the ordering condition becomes $X^{(1)} \preceq_{cx} X^{(2)}$.*

Proof. Obviously, the conditions $\lambda_1 = \lambda_2$, $\eta_1 = \eta_2$ and $X^{(1)} \preceq_{Lorenz} X^{(2)}$ yield $X^{(1)}/c_1 \preceq_{cx} X^{(2)}/c_2$. This implies $X^{(2)}/c_2 \preceq_{icv} X^{(1)}/c_1$ and Proposition 1 then gives $\psi_1(0, t) \ge \psi_2(0, t)$ for all $t > 0$. In the particular case where $\mu_1 = \mu_2$, we get $c_1 = c_2$ and the result follows. □

As mentioned in the introduction, Michel (1987) proved that the more a claim size is variable, the more the ultimate ruin probability is large. Such an implication does not hold over a finite-time horizon. Indeed, when $u = 0$, Corollary 1 shows that, on the contrary, the more a claim size is variable, the more the finite-time ruin probability is small. This seems a priori counter-intuitive but a possible explanation is as follows. When $u = 0$, the ruin, if it occurs, is very likely to happen early, at a time when the reserve is still small in comparison with the mean μ of a claim. Now, if the claim size is more variable, it has a significant chance to be smaller than μ and not to cause ruin. If it is larger than μ, this will not much influence the risk of ruin because ruin will be mainly due to the mean μ.

Example 1. In the case where $X^{(1)}$ and $X^{(2)}$ are exponentially distributed, it comes

$$\begin{aligned} E[(X^{(j)}/c_j - x)_-] &= \frac{1}{\mu_j} \int_0^{c_j x} e^{-y/\mu_j} \left(x - y/c_j\right) dy \\ &= -\frac{\mu_j}{c_j} \left(1 - e^{-c_j x/\mu_j}\right) + x, \quad j = 1, 2. \end{aligned}$$

Thus, the condition $X^{(1)}/c_1 \preceq_{icv} X^{(2)}/c_2$ of Proposition 1 becomes

$$-\frac{\mu_1}{c_1}\left(1 - e^{-c_1 x/\mu_1}\right) + x \ge -\frac{\mu_2}{c_2}\left(1 - e^{-c_2 x/\mu_2}\right) + x \quad \text{for all } x \ge 0.$$

Since $\theta\left(1 - e^{-x/\theta}\right)$ is increasing in θ, this condition is satisfied when

$$\frac{\mu_1}{c_1} \le \frac{\mu_2}{c_2},$$

i.e. when $\lambda_1(1+\eta_1) \le \lambda_2(1+\eta_2)$. In the particular case where $\lambda_1 = \lambda_2$ and $\eta_1 = \eta_2$, it is interesting to notice that $X^{(1)}/c_1 \preceq_{icv} X^{(2)}/c_2$ and $X^{(2)}/c_1 \preceq_{icv} X^{(1)}/c_2$, which means that $\psi_1(0, t) = \psi_2(0, t)$ for all t.

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