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Efficient option risk measurement with reduced model risk

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ABSTRACT

Options require risk measurement that is also computationally efficient as it is important to derivatives risk management. There are currently few methods that are specifically adapted for efficient option risk measurement. Moreover, current methods rely on series approximations and incur significant model risks, which inhibit their applicability for risk management.

In this paper we propose a new approach to computationally efficient option risk measurement, using the idea of a replicating portfolio and coherent risk measurement. We find our approach to option risk measurement provides fast computation by practically eliminating nonlinear computational operations. We reduce model risk by eliminating calibration and implementation risks by using mostly observable data, we remove internal model risk for complex option portfolios by not admitting arbitrage opportunities, we are also able to incorporate liquidity or model misspecification risks. Additionally, our method enables tractable and convex optimisation of portfolios containing multiple options. We conduct numerical experiments to test our new approach and they validate it over a range of option pricing parameters.

1. Introduction and outline of the paper

Computationally efficient risk measures of options are of paramount importance to research and industry, especially with the progressive increase in options trading and hedging. The events of the global credit crisis and past financial crises have demonstrated the necessity for adequate option risk management and measurement; poor risk measurement and management can result in bankruptcies and threaten collapses of an entire finance sector (see Kabir and Hassan, 2005). This is further exacerbated by the nonlinear losses associated with options and low margin requirements for options trading, which magnify losses.

Recently, there has been substantial literature on risk theory and risk measures, yet these have generally focussed on assets (e.g. stocks and bonds) rather than derivatives. Consequently, there is very little literature on option specific risk measurement. In order to measure the risk associated with an option we require the option's loss distribution. For the purpose of this paper let $Z(t)$ denote the loss distribution associated with some asset or derivative. For example

$$Z(t) = C(0) - C(t),$$

where $C(0)$ and $C(t)$ represent the call option price at time now and time t respectively. We denote a risk measure by $\rho(.)$ and measuring risk by $\rho(Z)$.

As the option loss distribution is typically not available in a closed form solution, it must be obtained by Monte Carlo simulation. However, this can be computationally time consuming, even for the simplest option pricing models, because it requires computation of nonlinear functions (relating to the option pricing equation). Such long computation times are unsuitable for many financial applications e.g. high frequency trading. Consequently, this has led to the development of more computationally efficient methods of option risk measurement.

To improve the computation speed of option risk, the typical approach has been to apply some mathematical approximation to the option's loss distribution (e.g. Delta method). However, such computational improvements have been generally achieved at the cost of model risk, that is unforeseen losses associated with using a model e.g. calibration errors, implementation errors, etc. Since the purpose of such models is to measure or manage risk, such model risks defeat the purpose of the models and represents a significant issue.

Model risk is becoming increasingly important in risk management due to the increasing potential for it to cause significant losses; this has partly arisen due to the increasing reliance on models in the financial industry. For instance, model risk has been cited as a partial cause of the global financial crisis. Many institutions prefer to use models with lower model risk than models that are theoretically more consistent e.g. single factor interest rate models are preferred to multi-factor models due to their lower model risk. Although multi-factor models may be more realistic at explaining

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interest rate movements, they can result in higher estimation errors compared to single factor models.

In this paper we approach option risk measurement from a new direction. Rather than pursuing approximation methods (as has been done with prior methods for option risk measurement), we measure option risk using the risk of its equivalent replicating portfolio. This replicating portfolio method practically eliminates the requirement for calculating nonlinear operations for option risk and so provides faster computation times. Moreover, our replicating portfolio approach has lower model risk compared to competing computationally efficient option risk measurement methods. The replicating portfolio method does not admit arbitrage opportunities for portfolios containing put and call options (unlike other models), our method also has lower calibration risk, it can take into account liquidity risks and model misspecification, it can model the option risk of option portfolios without losing computational tractability and enables portfolio optimisation.

The outline of the paper is as follows: firstly we introduce option risk measurement and review current computationally efficient methods for measuring option risk. In the next section we then introduce our replicating portfolio approach to risk measurement. We then discuss the advantages of the replicating portfolio approach with respect to computational efficiency and model risk. We then conduct numerical experiments and finally end with a conclusion.

2. Introduction to option risk measurement and literature review

In this section we introduce and review the literature on risk measurement, model risk, and computationally efficient option risk measurement.

2.1. Risk measurement and model risk

A risk measure ρ is a function mapping Z to $\mathbf{R}$, that is

$$\rho : Z \to \mathbf{R}.$$

We denote measuring risk by $\rho(Z)$. A popular industry risk measure is VaR (see Szegö, 2005), that is $F(Z(t) \leq \text{VaR}) = \beta$, where $F(.)$ is the cumulative probability distribution function and β is a cumulative probability associated with threshold value VaR, on the loss distribution of $Z(t)$.

A significant milestone in risk measurement was achieved when (Artzner et al., 1997) proposed the coherency axioms: axioms that risk measures $\rho(.)$ should obey to correctly measure risk. The coherency axioms are included in the Appendix for reference and further discussions on risk measures can be found in Goovaerts et al. (2004) and references therein.

To measure option risk we apply some risk measure to the loss distribution governing $C(0) - C(\delta t)$, where $C(\delta t)$ is the option value at some future time step δt. Whereas for stocks it is possible to analytically model the loss distribution in order to apply some risk measure, this is typically not possible for option loss distributions. Consequently, the option loss distribution of $C(0) - C(\delta t)$ must be obtained by computational methods (such as Monte Carlo simulation) and therefore the key difficulty in option risk measurement resides in obtaining the loss distribution in a computationally efficient approach. Once this distribution is obtained, we can apply a risk measure $\rho(.)$ to this distribution. For example, the VaR risk measure would determine the value associated with a cumulative probability β.

Currently, all option risk methods achieve computational efficiency in speed of computation by allowing model risk to increase. Model risk is defined as the risk of working with a potentially incorrect model, which leads to unexpected losses. Examples of model risks that can be incurred are increased calculation error, increased calibration errors or violation of fundamental theorems in Finance e.g. Law Of Arbitrage (to be addressed in later sections).

Model risk is a key problem in Finance; model errors can result in significant losses (e.g. Long Term Capital Management), they are playing an increasingly important role in industry and institutions are becoming ever more reliant on models for a variety of purposes. In option risk models, model risk is a particularly important issue because such models are used for risk management purposes. Hence it is important that such models have low model risks to prevent the models themselves incorrectly measuring risk or becoming a source of risk in themselves.

To give an example of model risk, the Delta–Gamma method (to be discussed later) should be theoretically always more preferable to the Delta method (to be discussed later) in calculating option risk. The Delta–Gamma method is a theoretically more accurate method than the Delta method, however the Delta–Gamma method requires calculation of European option parameter γ. As γ may not be available in analytic form for many option pricing models, it can only be calculated by computational methods, which can distort calculation accuracy but also increase total computation time. In fact it should be noted that computationally evaluating second order partial differential equations in general (such as γ) can be inaccurate. Hence the model risk (and computational efficiency) of the Delta–Gamma method will be worse than the Delta method. Furthermore, the Delta–Gamma method removes the linear relation between the change in stock price δS and change in call option price δC (see later sections for more details), which significantly complicates valuing portfolios with options and portfolio optimisation (unlike in the Delta method).

The current literature on model risk is limited in finance. In Kerkhof et al. (2010), model risk is taken into account to determine capital reserves for banks. In particular, estimation risk, identification and misspecification models risks are addressed and combined with standard risk measures such as VaR. In Kondo and Saito (2012), a Bayesian method is proposed for measuring model risk for the insurance loss ratio. This method makes specific distribution assumptions and is focussed around VaR calculations, rather than application to any specific risk measure. In Alexander and Sarabia (2012) they develop a method for calculating model risk with respect to quantile risk measurement only. This allows institutions to adjust capital reserves to meet potential losses arising from model risk. Schmeiser et al. (2012) analyse model risk with respect to solvency measures in the insurance sector.

Although there exists literature on model risk, the literature on model risk and computationally efficient option risk methods is non-existent to the best of our knowledge. The closest literature to address model risk with respect to option risk measurement is in Guillaume and Schoutens (2012), where model risk is investigated specifically with respect to calibration risk for vanilla and exotic options. However no reference is made with respect to computationally efficient option risk methods.

2.2. Option risk measurement

The current literature on option risk measurement is limited, particularly for computationally efficient methods. The most direct or "brute-force" approach to option risk measurement is the "full valuation method" (see Christoffersen, 2003). This involves Monte Carlo simulation of $S_i(\delta t)$ using some stock price model (e.g. geometric Brownian motion), where i denotes the index of the simulation sample. The option price value associated with $S_i(\delta t)$, that is $C_i(\delta t)$, is then calculated. The algorithm for the full valuation method is given in the Appendix for the Black–Scholes option pricing model $C(S(t), t, T, r, \sigma, K)$, which is also defined in the Appendix.