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Valuation of guaranteed minimum maturity benefits in variable annuities with surrender options[*]

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Abstract

We present a numerical approach to the pricing of guaranteed minimum maturity benefits embedded in variable annuity contracts in the case where the guarantees can be surrendered at any time prior to maturity that improves on current approaches. Surrender charges are important in practice and are imposed as a way of discouraging early termination of variable annuity contracts. We formulate the valuation framework and focus on the surrender option as an American put option pricing problem and derive the corresponding pricing partial differential equation by using hedging arguments and Itô's Lemma. Given the underlying stochastic evolution of the fund, we also present the associated transition density partial differential equation allowing us to develop solutions. An explicit integral expression for the pricing partial differential equation is then presented with the aid of Duhamel's principle. Our analysis is relevant to risk management applications since we derive an expression of the delta for the sensitivity analysis of the guarantee fees with respect to changes in the underlying fund value. We provide algorithms for implementing the integral expressions for the price, the corresponding early exercise boundary and the delta of the surrender option. We quantify and assess the sensitivity of the prices, early exercise boundaries and deltas to changes in the underlying variables including an analysis of the fair insurance fees.

JEL Classification: C63, G12, G22, G23

Keywords: Guaranteed minimum maturity benefits, Surrender options, Numerical integration