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Review

Sequential Monte Carlo Samplers for capital allocation under copula-dependent risk models

Rodrigo S. Targino [a,*], Gareth W. Peters [a], Pavel V. Shevchenko [a,b]

[a] *University College London, London, UK*

[b] *CSIRO, Sydney, Australia*

HIGHLIGHTS

- The paper studies the problem of capital allocation for copula-dependent risks.
- Under the Euler principle, allocations can be calculated as expectations conditional to a rare-event.
- We design a specialized Sequential Monte Carlo algorithm to calculate conditional expectations.
- Theoretical results (such as the relative error and the asymptotic variance) of the estimators are discussed.
- We show the efficiency of the algorithm when compared with simple Monte Carlo and Importance Sampling approaches.

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ABSTRACT

In this paper we assume a multivariate risk model has been developed for a portfolio and its capital derived as a homogeneous risk measure. The Euler (or gradient) principle, then, states that the capital to be allocated to each component of the portfolio has to be calculated as an expectation conditional to a rare event, which can be challenging to evaluate in practice. We exploit the copula-dependence within the portfolio risks to design a Sequential Monte Carlo Samplers based estimate to the marginal conditional expectations involved in the problem, showing its efficiency through a series of computational examples.

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* Corresponding author.
E-mail address: r.targino.12@ucl.ac.uk (R.S. Targino).

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1. Introduction

Since financial institutions are in the business of managing and reallocating risks, an important part of their internal risk management is to have an appropriate level of capital as a buffer against unexpected losses. Typically, retail banks, investment banks and insurance companies must satisfy their local jurisdiction version of capital adequacy, which are usually specified by the local regulatory authorities according to some version of either the Basel II/III banking supervision guides or the Solvency II insurance guides.

In this context capital may refer to two different quantities: Economic Capital (EC) or Regulatory Capital (RC). The first is the capital that would have be chosen in the absence of regulation. It represents the amount the institution estimates in order to remain solvent at a given confidence level and fixed time horizon and is set in order to meet some credit risk rating. The Regulatory Capital, in turn, reflects the needs given by regulatory guidance and rules. It is important to note that the capital actually been held by the institution (henceforth referred to as *capital*) will always be the maximum between the EC and the RC.

Both for banks and insurance companies, regulation has evolved towards Regulatory Capital based on risk measures (see Section 2). For banks, the Basel II accord set the standard to the Value at Risk (still in use at Basel III) while insurance directives such as Solvency II and the Swiss Solvency Test diverge on the risk measure to be used (the first suggests the usage of Value at Risk and the former Expected Shortfall). It is important to note that although the RC and the EC will differ in most of the financial institutions, both quantities are usually based on the same class of risk measures, differing only on the confidence level.

This work is focused on studying the problem of capital allocation, with particular focus on Operational Risk (OpRisk) capital—see Shevchenko (2011) for an text-book introduction to OpRisk modelling. In order to solely study this aspect, we will assume that the parametric risk models have been selected and the parameter estimations performed in each business unit or division for all the relevant risk types. Then using these models the bank or insurance company has obtained an estimate of the total capital from the model. Based on this capital figure, our work aims to study a problem that follows the calculation of the capital to be held, namely the second order problem of the capital allocation to different divisions and business units as a capital charge (see Table 1.1). Once the overall capital is calculated, the financial institution faces the problem of how to allocate this given capital among different risk sources, in order to understand how much each risk cell contributes to the total risk (capital) and in order to assess their risk management controls and performance, as part of the process discussed in the Pillar III of Basel II/III. Put another way there is a capital charge that must be allocated to each division and business unit which must reflect commensurately the risk profile of the given business unit. This continues to provide an incentive for banks following the Advanced Measurement Approach (AMA) to carefully model their dependence structures.

Apart from the fact that losses in some of the these risk cells may be dependent, the Basel II accord (BCBS, 2006) §657 ensures that the capital estimate can have diversification benefits if dependency modelling is approved by the local regulator. In other words, the bank may be authorized to set aside less Regulatory Capital if they can demonstrate evidence for dependence features in their loss processes between each business line or between risk types within a business line.

We will also assume the dependency among all risk cells in the portfolio is known from the first phase of model selection and estimation. More precisely, we will assume that the bank's portfolio consists of d individual losses (in a risk cell level) denoted by $X_1, \ldots, X_d$, each one modelled as random variables (rv's) with continuous cumulative distribution function (cdf) given by $F_i,\ i = 1, \ldots, d$.

The dependence structure of the losses will be given by a (known) copula $C(u_1, \ldots, u_d)$ (see Appendix B for the definition and some results regarding copulas), leading to a joint distribution of the losses given by

$$F_{\mathbf{X}}(\mathbf{x}) = C\big(F_1(x_1), \ldots, F_d(x_d)\big),$$

where $\mathbf{X} = (X_1, \ldots, X_d)$, and $\mathbf{x} = (x_1, \ldots, x_d)$.

In the recent years many academic works were devoted to the joint modelling of operational losses and it impact on capital calculation. Recently, Brechmann et al. (2014) introduced a zero-inflated dependence model, which is then coupled using different copulas (Archimedean, elliptical, individual Student's t and vine). Previously, Giacometti et al. (2008) used α-stable marginal distributions and Student's t copulas (both symmetric and skewed