Insurance: Mathematics and Economics 61 (2015) 36–47

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Assessing the solvency of insurance portfolios via a continuous-time cohort model[☆]

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HIGHLIGHTS

- We set up a continuous-time ALM model of an insurance portfolio.
- Liabilities are evaluated at fair-value, following recent regulatory trends.
- We analyze the effectiveness of natural hedging strategies.
- Natural hedging reduces risk when interest-rate risk is simultaneously managed.

ARTICLE INFO

Article history:
Received July 2014
Received in revised form November 2014
Accepted 3 December 2014
Available online 12 December 2014

JEL classification:
G22
G32

Keywords:
Longevity risk
Natural hedging
Continuous-time cohort models for longevity
Solvency of insurance portfolios
Solvency requirements
Longevity and interest-rate risk

ABSTRACT

This paper evaluates the solvency of a portfolio of assets and liabilities of an insurer subject to both longevity and financial risks. Liabilities are evaluated at fair-value and, as a consequence, interest-rate risk can affect both the assets and the liabilities. Longevity risk is described via a continuous-time cohort model. We evaluate the effects of natural hedging strategies on the risk profile of an insurance portfolio in run-off. Numerical simulations, calibrated to UK historical data, show that systematic longevity risk is of particular importance and needs to be hedged. Natural hedging can improve the solvency of the insurer, if interest-rate risk is appropriately managed. We stress that asset allocation choices should not be independent of the composition of the liability portfolio of the insurer.

1. Introduction

The assets and liabilities owned by insurance companies and pension funds are subject to various sources of uncertainty, making the assessment of their risk profile and solvency a challenging task. Regulators – for example, through the Solvency II Directive – are aiming at steering insurance companies towards a comprehensive accounting of the risks affecting their portfolios. This increasing attention to the soundness of risk management practices is enhancing the level of complexity of required valuation models, particularly in the context of the Own Risk Solvency Assessment (ORSA) process.

A proper assessment of the solvency of a portfolio requires the modeling of many risk sources. As companies invest in bonds and in the stock market, equity risk, together with interest-rate risk, affects the asset side. On the liabilities side, regulation in the Solvency II framework and the recent International Accounting Standards (IAS) have boosted the importance of market fair-valuation. From a risk management perspective, this entails both longevity

☆ The authors would like to thank Tom Hurd, Hong Li, Stephane Loisel, Elisa Luciano, John Michael Steele, Elena Vigna, an anonymous referee, seminar participants at McMaster University and conference participants at the Afmath Conference 2014 (Brussels), the Netspar International Pension Workshop (Venice), the 8th Conference in Actuarial Science and Finance (Samos), the First International Conference on Actuarial Science and Quantitative Finance (Bogotá) for their helpful comments.

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http://dx.doi.org/10.1016/j.insmatheco.2014.12.002
0167-6687/

risk and interest-rate risk assessment. The recent population aging phenomenon has clearly highlighted the exposure of annuity providers and life insurers to the uncertainty in mortality rates themselves (systematic longevity risk), coupled with the well-known randomness in the deaths of the policyholders in the portfolio (idiosyncratic longevity risk). Longevity risk, in both these dimensions, needs to be taken into account. It represents a relevant threat to the solvency of annuity providers and the hedging of its undiversifiable component has recently been investigated in the literature. Interest-rate risk impacts the value of liabilities, as they need to be discounted using the current term structure. As a consequence, the overall risk profile of the company is influenced by both the asset allocation strategy and the liability portfolio composition, and the choices regarding these two dimensions are deeply interconnected. However, in practice, quite surprisingly, liability hedging is still widely neglected. A recent Mercer (2013)'s survey highlights that only 26% of pension fund managers in the sample perform LDI (Liability Driven Investment) strategies of any kind, and that longevity risk is rarely managed.

This paper highlights the importance of managing longevity risk by assessing its relevance in an annuity portfolio. We explore the effectiveness of so-called natural hedging strategies, which mitigate systematic longevity risk by mixing annuities and life insurance policies. We focus on natural hedging as it could constitute a readily available and feasible alternative to the use of mortality derivatives, whose market, albeit slowly expanding (Blake et al., 2014), is still lacking in volume and standardization. While hedging strategies using derivative products are very effective in theory (see Ngai and Sherris (2011), for instance), the lack of liquidity that their market experiences may cause adverse selection problems (Biffis and Blake, 2013) or inefficiencies (Luciano and Regis, 2014).

Natural hedging strategies of longevity risk have recently been studied. Cox and Lin (2007) first documented that insurance companies that mixed annuities and life insurance policies experience a comparative advantage with respect to annuity-only providers. On these grounds, and given that natural hedging is easy to implement and cheap to insurance companies, the academic literature has recently explored the implementation and effectiveness of such strategies. Wang et al. (2010) and Wang et al. (2013) developed immunization strategies, where mortality is described by means of discrete-time models. Gatzert and Wesker (2012) numerically analyzed the potential risk mitigation provided by the liability mix, under different investment strategies, finding that the overall risk of a company can be reduced considerably. These works, however, do not evaluate liabilities at fair-value. Stevens et al. (2011) highlighted the importance of considering the interactions between longevity risk and financial risks, as such an omission might lead to overestimation of the natural hedging potential. Luciano et al. (2012) proposed a Delta–Gamma hedging strategy, accounting for the effects of natural hedging on the interest-rate risk exposure of the company.

This paper is the first to propose an analysis of the effectiveness of natural hedging strategies in the context of an ALM model of the insurance company in which liabilities are evaluated at market values and affected by interest-rate and longevity risks alike. We couple a standard description of the financial market by means of the well-known Vasicek (1977) model with a parsimonious description of mortality risk via a continuous-time cohort based stochastic model, following Luciano and Vigna (2008). This choice, in addition to being reasonably accurate in describing the evolution of mortality and interest rates, allows us to obtain the fair-value reserves of liabilities and their sensitivities (Greeks) to relevant risk factors in closed form (Luciano et al., 2012). This permits us to account for multiple risk sources, while considerably reducing the computational effort. Our analysis extends Hari et al. (2008), who focused – as we do – on the characteristics of the funding ratio of annuity providers. Apart from selecting a different mortality modeling strategy, we complement their analysis by introducing interest-rate risk uncertainty in our simulations and accounting for the presence of life insurance policies on the side of the liabilities.

Our numerical analysis, calibrated on UK data, allows us to assess the impact of the liability mix, together with the asset mix, on the solvency and bankruptcy likelihood of a portfolio of insurance policies in run-off. First, it documents the relevant impact of systematic longevity risk on annuity portfolios. While interest-rate risk is the most relevant risk source at short horizons, systematic longevity risk largely affects the variability of portfolio value in the medium and long run and needs to be managed for solvency purposes. Second, we analyze the effects of natural Delta-hedging strategies as proposed by Luciano et al. (2012). They are effective in reducing longevity risk and in improving the solvency of an annuity portfolio, especially when it is well-diversified (i.e. large enough). When the additional interest-rate risk due to the introduction of the portfolio of life insurance policies is not hedged, the company can worsen its risk profile and experience higher bankruptcy likelihood in the long run. We thus highlight the importance of jointly determining asset allocation and liability mix choices.

The paper is organized as follows. In Section 2, we present our framework and describe our modeling of the risk sources. In Section 3, we present numerical results from our simulations, based on a calibrated example given the relevant UK data. Finally, in Section 4 we make conclusions and propose further research.

2. Setup

In order to properly provide an assessment of the risk profile of an insurance portfolio, it is necessary to have a comprehensive view of the risks surrounding its assets and liabilities, both concerning demographic and financial aspects. In this section, we describe an asset–liability model of a company, including

1. **interest-rate risk**, due to the stochastic fluctuations of the short rate;
2. **idiosyncratic longevity risk**, due to the uncertainty in the death arrival times of the individuals;
3. **systematic longevity risk**, due to the unexpected changes in the mortality intensity of the pool of policyholders, and
4. **equity risk**, due to the investment in the stock market.

While equity risk affects the asset side of the portfolio only – assuming that no participating policies are issued – the first three risk sources affect both the assets and liabilities, when the latter is evaluated on a market-consistent basis. Each of these risk sources is described by continuous-time stochastic processes, which we appropriately discretize when simulating. Discretization is done at intervals of time-length Δ, such that $t_i = t_0 + i\Delta,\ i \in \mathbb{I} = \{1, 2, \dots, N_0 \in \mathbb{N}\}$.

While, in principle, we can have dependence between financial risks and longevity risk (see Jalen and Mamon (2009)),[1] we follow the most common approach in the literature and assume their independence. This assumption is reasonable, at least in the short run, as Cairns et al. (2006) point out. In addition, given our modeling choices, independence under historical measure translates to independence under the pricing measure, a result which need not hold in general (see Dhaene et al. (2013)).

2.1. Liabilities

The liability portfolio of the insurer is composed of standard insurance policies: whole-life annuities (A) and temporary death contracts (D). We focus on a portfolio made by a homogeneous

[1] Although the assumption of independence does prove to be convenient from a computational point of view, our framework can easily accommodate dependence as well.