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In-sample forecasting applied to reserving and mesothelioma mortality

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HIGHLIGHTS

- Feasible forecasting methods from data only available at historical calendar times.
- Applications to stochastic claims reserving and asbestos mortality forecasting.
- Finite sample simulations under scenarios close to real life problems.
- Novel general asymptotic theory for the proposed methods.

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ABSTRACT

This paper shows that recent published mortality projections with unobserved exposure can be understood as structured density estimation. The structured density is only observed on a sub-sample corresponding to historical calendar times. The mortality forecast is obtained by extrapolating the structured density to future calendar times using that the components of the density are identified within sample. The new method is illustrated on the important practical problem of forecasting mesothelioma for the UK population. Full asymptotic theory is provided. The theory is given in such generality that it also introduces mathematical statistical theory for the recent continuous chain ladder model. This allows a modern approach to classical reserving techniques used every day in any non-life insurance company around the globe. Applications to mortality data and non-life insurance data are provided along with relevant small sample simulation studies.

1. Introduction

Let us assume that we have a structured density defined as a density that is a known function of one-dimensional densities, see Mammen and Nielsen (2003) for the equivalent definition of structured regression. Assume furthermore that observations are available from this structured density on a restricted support only. Finally assume that the character of this restricted support is such that in-sample information is available for all the one-dimensional functions defining the original structured density. In this situation, an extrapolation or forecast is immediately available for that part of the support without observations. It turns out that one of the most important problems in non-life insurance, estimation of outstanding liabilities in reserving, has exactly this form. The structured density is most often a multiplicative density in this case. The support with observations represents insurance claims until the current calendar time, and the support without observations represents future insurance claims. This forecast method has traditionally been called the chain ladder technique in actuarial science and the multiplicative density has been estimated as a structured histogram or equivalently from maximum likelihood assuming a multiplicative Poisson structure, see Wüthrich and Merz (2008) for an overview and Kuang et al. (2009), Verrall et al. (2010), Martínez-Miranda et al. (2011, 2012, 2013a,b,c), for recent reformulations of classical chain ladder in mathematical statistical terms published in the actuarial literature. Other recent contributions in reserving considering statistical models based on individual claims include Antonio and Plat (2014) and Pigeon et al. (2013, 2014). The

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longevity problem is another important application of structured density forecasting and as in non-life insurance a histogram type of approach is widely used and analyzed, see Haberman and Renshaw (2012) and Hatzopoulos and Haberman (2013). In this paper we propose to use our alternative approach based on structured non-parametric models and we illustrate its power by applying it to mesothelioma mortality forecasts. We compare our empirical findings with Martínez-Miranda et al. (2015) who used a classical approach based on a multiplicative Poisson structure.

While we stick to the multiplicative density structure in this paper, it is evident that important generalizations are possible. One could add a variety of one-dimensional densities to the overall structure leading to non-multiplicative structures. It would also be interesting to generalize the approach of this paper to other sources of mortality than age and cohort. One example would be to add calendar time effects generalizing the histogram approach to calendar effect estimation developed in Kuang et al. (2008a,b). Another would be to add time independent or time dependent covariates. It would be also interesting to consider the work of Zhang et al. (2013) to develop distribution free prediction sets, see Lei et al. (2013). Finally, the approach of projecting a multivariate density smoother down on the structure of interest is not restricted to local linear density smoothers and could be generalized to other multivariate density smoothers including Panaretosa and Konis (2012), Xiao et al. (2013) and Lu et al. (2013).

The paper is organized as follows. In Section 2 the structured density model is formulated in the special multiplicative case. A projection approach based on local linear density estimation is defined. The asymptotic properties of the suggested method are provided in Section 6 (with more details and proofs deferred to Appendix). Applications to non-life insurance and mesothelioma mortality forecasting are explained in Section 3. While these two applications rely on the multiplicative density structure, observations are available on very different underlying supports. However, for both applications the entering one-dimensional densities are identified by the observed data. The analyses of two datasets are described in Section 4. Section 5 includes a brief simulation study with simulation settings defined to be close to real life situations. All the calculations in the paper have been performed with R, (R Development Core Team, 2014).

2. Multiplicative density forecasting

2.1. Model formulation

Let us consider n i.i.d. observations $\{(X_i, Y_i), i = 1, \ldots, n\}$ from a two-dimensional variable (X, Y) having a density f with support on a subset $\mathcal{I}_f$ of the rectangle $\mathcal{S}_f = \{(x, y) : 0 \le x \le T_1, 0 \le y \le T_2\}$, with $T_1, T_2 > 0$. The aim is to forecast the density of (X, Y) in $\mathcal{S}_f$ from the given observations that are only available in the set $\mathcal{I}_f$. To this goal let us assume that f is multiplicative, i.e., it is of the form:

$$f(x, y) = c_f f_1(x) f_2(y), \tag{1}$$

where f_1 and f_2 are probability densities on $[0, T_1]$ and $[0, T_2]$, respectively. The constant c_f is chosen such that

$$\int_{\mathcal{I}_f} c_f f_1(x) f_2(y) dx\, dy = 1. \tag{2}$$

This formulation transforms the original forecasting problem to an estimation problem of the densities f_1 and f_2. The approach of this paper is developed for a general support $\mathcal{I}_f$ including the two different support structures that came up in our two applications (mortality studies and insurance reserving). See also Nielsen and Linton (1998) for related projection methods in structured non-parametric regression.

Note that if the support where the densities are observed is a rectangle, then the estimation problem would be trivial and both components could be estimated separately. The non-rectangular supports considered in this paper imply that the estimation problem is more complicated. However, we are only considering non-rectangular supports, where the multiplicative components are still estimable in-sample. While the term in-sample forecasting is defined in this paper, the in-sample forecasting trick is an old one and has been used in non-life insurance in actuarial science as long as anyone remembers. In actuarial science the non-rectangular support is a triangle and the multiplicative structure is estimated via a parametric approach related to maximum-likelihood estimation, see Kuang et al. (2009). It has recently been pointed out that this classical actuarial forecasting methodology can be understood as first estimating a multiplicatively structured histogram and then extrapolating into the future, see Martínez-Miranda et al. (2013a). More complicated structures violating the independence assumption between X and Y could also be considered. This is, however, beyond the scope of this paper. Among many examples one could imagine that a calendar time effect enters the model in some multiplicative way, see Kuang et al. (2011) for a classical structured histogram approach to forecasting including such a calendar effect.

2.2. The projection approach

Consider the density f with support $\mathcal{I}_f$ and consider one point $(x, y) \in \mathcal{I}_f$. The local linear estimator introduced in Nielsen (1999) and Müller and Stadtmüller (1999) is derived by solving the following minimization problem:

$$\widehat{\Theta} = \arg\min_{\Theta} \lim_{b\to 0} \int_{\mathcal{I}_f} \left\{\widetilde{f}_b(u, v) - \theta_1 - \theta_{2,1}(u - x) - \theta_{2,2}(v - y)\right\}^2 \times K_{h_1}(u - x) K_{h_2}(v - y) du\, dv, \tag{3}$$

where $\Theta = (\theta_1, \theta_{2,1}, \theta_{2,2})$, $\widehat{\Theta} = (\widehat{\theta}_1, \widehat{\theta}_{2,1}, \widehat{\theta}_{2,2})$ and $\widetilde{f}_b(u, v) = n^{-1}\sum_{i=1}^n K_b(X_i - u) K_b(Y_i - v)$. Here $K_b(u) = b^{-1} K(u/b)$, for a one-dimensional symmetric kernel function K and bandwidth parameters $b > 0$, $h_1 > 0$, $h_2 > 0$. The local linear density of $f(x, y)$ is given by $\widehat{f}_{h;\mathcal{I}_f}(x, y) = \widehat{\theta}_1$, which is defined for any given vector of bandwidth parameters $h = (h_1, h_2) \in \mathbb{R}_+^2$.

Note that $\widehat{f}_{h;\mathcal{I}_f}(x, y)$ is an estimator of the density f of (X, Y) restricted to the support $\mathcal{I}_f$. Forecasting into the future amounts to extrapolating our estimated density to the full support $\mathcal{S}_f$. This forecast or extrapolation is only possible under some assumptions on the functional form of $f(x, y)$. In this paper, we consider one of the simplest structured density options (1) and project the unrestricted local linear estimator down on the relevant multiplicative space. Specifically c_f, f_1 and f_2 are estimated by minimizing the following expression:

$$\min_{c_f, f_1, f_2} \int_{\mathcal{I}_f} \left(\widehat{f}_{h;\mathcal{I}_f}(x, y) - c_f f_1(x) f_2(y)\right)^2 w(x, y) dx\, dy, \tag{4}$$

under the constraint that $\int_0^{T_1} f_1(x)dx = 1$ and $\int_0^{T_2} f_2(y)dy = 1$. Here w is some weighting function such that $w(x, y) > 0$.

In practice, the above minimization can be done by using the following iterative algorithm:

1. Consider the estimator $\widehat{f}_{h;\mathcal{I}_f}$ derived above and $\widehat{f}_{1,h}^{(0)}$ being an initial estimator of f_1.
2. Calculate an estimator the f_2 as

$$\widehat{f}_{2,h}^{(1)}(y) = \frac{\int_{\mathcal{I}_y} \widehat{f}_{h;\mathcal{I}_f}(x, y) w(x, y) dx}{\int_{\mathcal{I}_y} \widehat{f}_1^{(0)}(x) w(x, y) dx},$$

where $\mathcal{I}_y = \{x : (x, y) \in \mathcal{I}_f\}$.