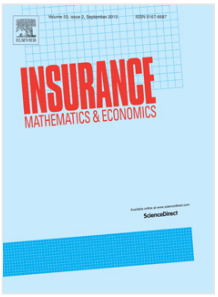

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Ghobad Barmalzan, Amir T. Payandeh Najafabadi, Narayanaswamy Balakrishnan

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Stochastic comparison of aggregate claim amounts between two heterogeneous portfolios and its applications

Ghobad Barmalzan[a], Amir T. Payandeh Najafabadi[a] and Narayanaswamy Balakrishnan[b,c]

[a] Department of Mathematical Sciences, Shahid Beheshti University, G.C. Evin, Tehran, Iran

[b] Department of Mathematics and Statistics, McMaster University, Hamilton, Canada

[c] Department of Statistics, King Abdulaziz University, Jeddah, Saudi Arabia

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Abstract

The aggregate claim amount in a particular time period is a quantity of fundamental importance for proper management of an insurance company and also for pricing of insurance coverages. In this paper, we show that the proportional hazard rates (PHR) model, which includes some well-known distributions such as exponential, Weibull and Parato distributions, can be used as the aggregate claim amount distribution. We also present some conditions for the use of exponentiated Weibull distribution as the claim amount distribution. The results established here complete and extend the well-known result of Khaledi and Ahmadi (2008, *Journal of Statistical Planning and Inference*, **138**, 2243-2251).

Keywords: Aggregate Claim Amount; Exponentiated Weibull Distribution; Pareto Distribution; Multivariate Chain Majorization; Usual Multivariate Stochastic Order; Proportional Hazard Rates Model.

1 Introduction

The dynamics of insurance industry has different effects on the number of claims and amount of claims. For instance, expanding insurance business would have proportional increase on number of claims but may have negligible impact on amount of claims. Conversely, cost control initiatives and technology innovations would have adverse effect on amount of claims but may not have any effect on the number of claims. For these reasons, the aggregate claim is modeled with the assumption that the number of claims and the amount of claims are independent.

Throughout, we have considered the following heterogeneous portfolios: $(I_{p_1}X_{\lambda_1},\cdots,I_{p_n}X_{\lambda_n})$, where $X_{\lambda_1},\cdots,X_{\lambda_n}$ are the claim sizes for the different risks, and n is the total number of possible claims in the portfolio. With each risk $i=1,\cdots,n$, we associate a Bernoulli random variable I_{p_i} to indicate whether the claim actually occurs or not. Thus, $S_n(\boldsymbol{\lambda},\boldsymbol{p})=\sum_{i=1}^{n}I_{p_i}X_{\lambda_i}$ is the total claim size for this portfolio. In particular, in actuarial science, it corresponds to the aggregate claim amount in a portfolio of risks.

Stochastic orders have been used in various areas including management science, financial economics, insurance, actuarial science, operations research, reliability theory, queuing theory, and survival analysis. For example, in the decision theory of financial economics, they can help an individual to make suitable decisions by comparing pairs of risks leading to different uncertain payments. Interested readers may refer to Müller and Stoyan (2002)