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Note on the Hybrid Flowshop Scheduling Problem with Multiprocessor Tasks

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ABSTRACT

In this note the Hybrid Flowshop Scheduling Problem with Multiprocessor Tasks is addressed. The objective function to be minimized is the maximum completion time or the makespan. The main purpose of this note is to pinpoint an inaccuracy contained in a recent paper while developing a lower bound (Chou: IJPE, 141:137–145) and to propose some valid lower bounds.

1. Introduction

The *Hybrid Flowshop Scheduling Problem* with *Multiprocessor Tasks* (HFSMT) is stated as follows. A set $J = \{1, 2, \ldots, n\}$ of n jobs has to be treated on s production centers $Z_1, Z_2, \ldots, Z_s$ in that order. The processing time of job $j \in J$ on center Z_k $(k = 1, \ldots, s)$ is p_{kj}. Each stage Z_k is composed of m_k parallel and identical machines. The processing of a job $j \in J$ in center Z_k $(k = 1, \ldots, s)$ requires $seiz_{kj}$ simultaneous machines. During the processing phase the following constraints should be respected. Each machine can process at most one job at any time and the preemption is not allowed. All jobs and all machines are available from time zero. In addition, all the processing times p_{ij} and the required machines $size_{ij}$ $(i = 1, \ldots, s$ and $j \in J)$ are integer and deterministic. The buffer capacity between the centers is assumed to be infinite. The objective is to provide a feasible schedule that minimizes the maximum completion time, or makespan. When $size_{ij} = 1 (i = 1, \ldots, s$ and $j \in J)$ the HFSMT is reduced to the hybrid flow shop scheduling problem which is an interesting and challenging problem (Omid and Rasaratnam, 2016).

During the last decade, the (HFSMT) have been investigated in the scheduling literature. For a comprehensive surveys the reader is referred to Ribas et al. (2010) and Ruiz and Vázquez-Rodríguez (2010). Recent contributions are the papers of Lahimer et al. (2011), Lahimer et al. (2013), Chou (2013) and Lin et al. (2013).

In this note, we proof that a lower bound recently proposed in Chou (2013) is incorrect and we present a new valid lower bounds.

2. Lower bounds for the Hybrid Flowshop Scheduling Problem with Multiprocessor Tasks

2.1. The lower bound of Chou (2013)

In this lower bound the set of jobs J for each center Z_i $(i = 1, \ldots, s)$ is partitioned into the following subsets:

- $E_i = \{j \in J\colon size_{ij} = m_i\}$
- $F_i = \left\{j \in J\colon size_{ij} > \frac{1}{2}m_i \text{ and } size_{ij} = m_i - 1\right\}$
- $G_i = \left\{j \in J\colon \frac{1}{2}m_i < size_{ij} < m_i - 1\right\}$
- $H_i = \left\{j \in J\colon size_{ij} = \frac{1}{2}m_i\right\}$
- $L_i = \left\{j \in J\colon 1 < size_{ij} < \frac{1}{2}m_i\right\}$
- $Q_i = \{j \in J\colon size_{ij} = 1\}$

In addition, the following notations are introduced.

$$\begin{aligned} bp_i(1) &= \tfrac{1}{m_i}\textstyle\sum_{j=1}^{n} p_{ij} \times size_{ij}, \\ bp_i(3) &= \textstyle\sum_{j\in E_i\cup F_i\cup G_i} p_{ij} + \left\lceil \tfrac{1}{2}\textstyle\sum_{j\in H_i} p_{ij}\right\rceil \\ &+ \tfrac{1}{m_i}\left\{ Max\,[\,0,\ Max\left(0,\ \textstyle\sum_{j\in Q_i} p_{ij} - \textstyle\sum_{j\in F_i} p_{ij}\right)\right. \\ &+ \left.\textstyle\sum_{j\in L_i} p_{ij} \times size_{ij} - m_i \textstyle\sum_{j\in G_i} p_{ij} + \textstyle\sum_{j\in G_i} p_{ij} \times size_{ij}\ ]\right\}. \end{aligned}$$

In (Chou, 2013), Chou claims that:

$$SB_C = \max_{i=1,\ldots,s}\left\{\min_{j=1,\ldots,n}\left(\sum_{l=1}^{i-1} p_{lj}\right) + \max(bp_i(1), bp_i(3)) + \min_{j=1,\ldots,n}\left(\sum_{l=i+1}^{s} p_{lj}\right)\right\}$$

is a lower bound for the HFSMT problem.

Actually, SB_C is not a valid lower bound. This can be proofed by the following example.

Example 1. Consider the instance with $n = 9$, $s = 2$, and $m_1 = m_2 = 6$. The processing times p_{ij} and the required machines $size_{ij}$ $(i = 1, \ldots, s$ and $j \in J)$ are displayed in Table 1.

the obtained subsets are:

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Table 1
Data of Example 1.

j	1	2	3	4	5	6	7	8	9
p_{1j}	3	3	3	3	3	3	3	3	3
$size_{1j}$	6	5	4	3	2	1	1	1	1
p_{2j}	3	3	3	3	3	3	3	3	3
$size_{2j}$	6	5	4	3	2	1	1	1	1

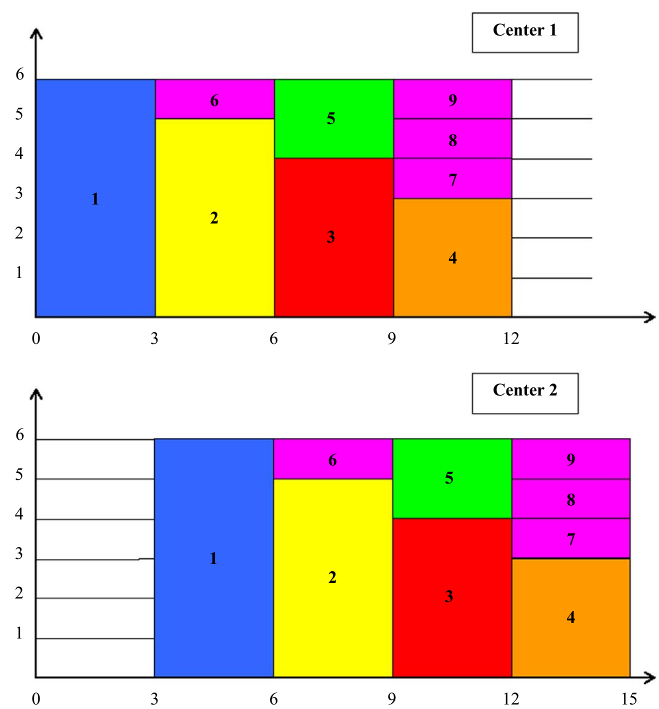

Fig. 1. Gantt chart of a feasible schedule having a makespan equal to 15.

Table 2
Data of Example 3 ($LB_1 > LB_{OL}$).

j	1	2	3	4	5	6	7
p_{1j}	1	1	1	1	1	1	6
$size_{1j}$	5	3	1	1	1	1	1
p_{2j}	1	1	1	1	1	1	6
$size_{2j}$	5	3	1	1	1	1	1

Table 3
Data of Example 4 ($LB_1 < LB_{OL}$).

j	1	2	3	4	5	6	7
p_{1j}	1	1	1	1	1	1	1
$size_{1j}$	4	3	1	1	1	1	1
p_{2j}	1	1	1	1	1	1	1
$size_{2j}$	4	3	1	1	1	1	1

Table 4
Data of Example 6 ($LB_2 > LB_{OL}$).

j	1	2	3	4	5	6	7	8	9	10
p_{1j}	3	3	3	3	1	1	1	1	1	1
$size_{1j}$	4	3	3	2	1	1	1	1	1	1
p_{2j}	3	3	3	3	1	1	1	1	1	1
$size_{2j}$	4	3	3	2	1	1	1	1	1	1

Table 5
Data of Example 7 ($LB_2 < LB_{OL}$).

j	1	2	3	4	5	6	7	8	9	10
p_{1j}	3	3	3	3	1	1	1	1	1	1
$size_{1j}$	2	2	2	2	1	1	1	1	1	1
p_{2j}	3	3	3	3	1	1	1	1	1	1
$size_{2j}$	2	2	2	2	1	1	1	1	1	1

- $E_1 = E_2 = \{1\}$.
- $F_1 = F_2 = \{2\}$.
- $G_1 = G_2 = \{3\}$.
- $H_1 = H_2 = \{4\}$.
- $L_1 = L_2 = \{5\}$.
- $Q_1 = Q_2 = \{6, 7, 8, 9\}$.

In addition, for $i = 1$, we have:

- $bp_1(1) = \frac{1}{m_1} \sum_{j=1}^{n} p_{1j} \times size_{1j} = \frac{1}{6}(3 \times 6 + 3 \times 5 + 3 \times 4 + 3 \times 3.$

$$+ 3 \times 2 + 3 \times 1 + 3 \times 1 + 3 \times 1 + 3 \times 1) = \frac{72}{6} = 12$$

- $\sum_{j \in E_1 \cup F_1 \cup G_1} p_{1j} = p_{11} + p_{12} + p_{13} = 3 + 3 + 3 = 9.$
- $\left\lceil \frac{1}{2} \sum_{j \in H_1} p_{1j} \right\rceil = \lceil \frac{1}{2} p_{14} \rceil = \lceil \frac{3}{2} \rceil = 2..$
-

$$Max\left(0, \sum_{j \in Q_1} p_{1j} - \sum_{j \in F_1} p_{1j}\right) = Max(0, (p_{16} + p_{17} + p_{18} + p_{19}) - p_{12}).$$

$$= Max(0, (3 + 3 + 3 + 3) - 3) = 9$$

-

$$\sum_{j \in L_1} p_{1j} \times size_{1j} - m_1 \sum_{j \in G_1} p_{1j} + \sum_{j \in G_1} p_{1j} \times size_{1j} \quad .$$

$$= p_{15} \times size_{15} - m_1 \times p_{13} + p_{13} \times size_{13} = 3 \times 2 - 6 \times 3 + 3 \times 4 = 0$$

Thus,

$$\frac{1}{m_1}\left\{Max\left[0, Max\left(0, \sum_{j \in Q_1} p_{1j} - \sum_{j \in F_1} p_{1j}\right) + \sum_{j \in L_1} p_{1j} \times size_{1j} - m_1 \sum_{j \in G_1} p_{1j} + \sum_{j \in G_1} p_{1j} \times size_{1j}\right]\right\} = \frac{1}{6}\{Max[0, 9]\} = \frac{9}{6} = 1.5.$$

Hence $bp_1(3) = 9 + 2 + 1.5 = 12.5$. Furthermore, we have

- $\min_{j=1,\dots,9}(\sum_{l=1}^{0} p_{1j}) = 0,$
- $\min_{j=1,\dots,9}(\sum_{l=2}^{2} p_{2j}) = \min(p_{21}, p_{22}, p_{23}, p_{24}, p_{25}, p_{26}, p_{27}, p_{28}, p_{29}) = 3,$
- $\min_{j=1,\dots,9}(\sum_{l=1}^{1} p_{1j}) = \min(p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}) = 3,$
- $\min_{j=1,\dots,9}(\sum_{l=2}^{2} p_{1j}) = 0$

For $i = 2$, the same calculation gives: $bp_2(1) = 12$ and $bp_2(3) = 12.5$. Thus, we get

$$SB_C = \max\{0 + \max(12, 12.5) + 3, 3 + \max(12, 12.5) + 0\} = 15.5$$

However, we have a feasible schedule, having a makespan equal to 15, which is depicted in Fig. 1.

The mistake for the lower bound of Chou (2013) is contained in the expression of $bp_i(3)$ $(i = 1, \dots, s)$. Indeed, omitting $bp_i(3)$ $(i = 1, \dots, s)$ from SB_C results in the following valid lower bound:

$$LB_p = \max \sum_{i=1,\dots,s} \left\{ \min_{j=1,\dots,n}\left(\sum_{l=1}^{i-1} p_{lj}\right) + bp_i(1) + \min_{j=1,\dots,n}\left(\sum_{l=i+1}^{s} p_{lj}\right)\right\}$$

More explicitly and according to Chou (2013) , For each center Z_i $(i = 1, \dots, s)$ we have:

- The jobs in $Q_i = \{j \in J: size_{ij} = 1\}$ are processed simultaneously with