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Dynamic capacity management with uncertain demand and dynamic price

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ABSTRACT

Inspired from practices of Chinese real estate firms, we study dynamic capacity management problems for a firm that sells homogeneous goods over a finite or infinite selling horizon, provided that demand is uncertain, retail price is exogenous but changes according to an auto-regressive process, and there is holding cost for remaining inventory in each period. The capacity problems facing the firm include to determine an initial stock at the beginning of the selling season and to allocate the remaining quantity for the current and future sale in each period to maximize the firm's total expected discounted profit. By using Markov decision processes, we show the optimality of a so-called save-up-to level policy (i.e., it is optimal to allocate as much as the save-up-to level to future periods if possible in each period) and the existence of the optimal initial stock. Moreover, we characterize monotone properties of the optimal save-up-to level and the optimal initial stock. Numerical analysis shows that compared to the policy with zero save-up-to level (i.e., always allocating all the remaining inventory to the current period), the advantage of the policy proposed in the present paper is significant when price rising trend is large, holding cost is low, initial price is high, or demand variation is either high or low.

1. Introduction

In this paper, we examine a dynamic capacity management problem for a firm who faces uncertain demand and sells a product over a finite/infinite horizon, provided that retail price in each period exogenously changes according to an auto-regressive process. The firm must determine a capacity (i.e., an initial stock) at the beginning of the selling horizon and allocate in each period the inventory for both the current and future sale over the horizon to maximize its profit.

Our work is motivated by two sides including business and the literature. From the business side, many real estate firms in China are highly interested in using discretionary sale to dynamically manage capacity over selling horizons. For example, when the market is hot (i.e., retail price is expected to rise in future), firms tend to purchase a large number of lands and build a large number of houses. Meanwhile, they take conservative policies of *cover plate* and are reluctant to sell out houses on hand.[1] Whereas, when the market is slack and the price continues to fall, firms tend to purchase a small number of lands or not to purchase any lands. Moreover, they keep lands undeveloped and try to sell out houses on hand.[2]

These behaviors of real estate firms in China are for two reasons. The first reason is because of rapid development of Chinese real estate industry in the last two decades. For example, the ratio of the real estate industry to GDP in China is about 14% in 2012. An emergent developing market is uncertain in many aspects including demand and market prices, which brings real estate firms with opportunities and also challenges. Hence, firms prefer dynamic capacity management under uncertain market environment to maximize their profits. The second reason is because of the inherent attributes of real estate products. For a given region, its real estate products (i.e., houses) are durable but non-replenished (D-NR) in a short term. These D-NR attributes differentiate real estate products from both traditional manufacturing products considered in supply chain management (which are

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[1] http://www.dfdaily.com/html/113/2013/2/25/951294.shtml (accessed on March 20, 2014).
[2] http://www.infzm.com/content/98459 (accessed on March 20, 2014).

durable and replenished) and perishable products such as airline tickets studied in revenue management (which are non-durable and non-replenished). As durable products, there is a holding cost for unsold inventory. In addition, houses have very long production leadtime and cannot be moved from one region to another region. That is, the quantity of houses is fixed once they are built in a specific region. Hence, a real estate firm must make right decision on the capacity (i.e., the initial stock) of houses. Finally, due to durable attribute of houses, firms are able to sell houses over a long horizon (which can be approximated by infinite horizon). Therefore, it is important for a real estate firm to rightly decide the initial stock and allocate inventory in each period to the current and future sale over the selling horizon.

From the literature side, to our knowledge, seat inventory control (i.e., seat allocation) and dynamic pricing are two main streams of research in revenue management but they are separately studied. The research on seat inventory control, pioneered by Littlewood (1972), has received a good deal of attention, see a survey paper (McGill and van Ryzin, 1999). Several common assumptions in the literature involve (1) potential buyers are exogenously segmented into several different classes, (2) each class is associated with a fixed exogenous price, and (3) potential buyers arrive sequentially according to (increasing fare) classes.

The research on dynamic pricing is involved in two aspects. The first one is continuous time dynamic pricing pioneered by Gallego and van Ryzin (1994), where potential buyers arrive according to a Poisson process and each buys at most one item. The second one is discrete time (i.e., periodical) dynamic pricing (Bitran and Mondschein, 1997), where one important assumption is that there is at most one customer to arrive within each period (one-arrival-per-period). However, the assumption on consumer's sequential arrival according to *increasing fare classes* seems unreasonable in most practical cases (such as purchase for concert or sports event tickets). Moreover, usually, there is a cost for adjusting prices. That is, continuous time dynamic pricing with high-frequency price adjustment may be very expensive. Hence, dynamic pricing in practice is often periodical and the period between two adjacent epochs for adjusting prices may be relatively long. This implies that it is also unreasonable to make the *one-arrival-per-period* assumption.

Therefore, we propose our research problem by combining some decisions with assumptions involved in seat inventory control and discrete time dynamic pricing. First, similar to seat inventory control research, the decision in each period is to allocate on-hand inventory for both the current and future sale before the current demand is realized. In addition, similar to dynamic pricing research, customers are neither segmented into several different classes nor distinguished for their arrival in different periods. Meanwhile, we abandon the one-arrival-per-period assumption. Second, similar to seat inventory control research, retail prices in different periods are exogenously given. However, we assume that retail prices dynamically change according to an auto-regressive process. In a highly competitive market, such as the real estate market in China, retail prices are determined by the market and change over time.

In this paper, we mainly study the optimal initial stock and capacity allocation policies for a firm (such as a real estate firm) that sells a product over a finite/infinite horizon. Before the selling horizon starts, the firm places an initial order for its sale over the whole horizon. In each period of the selling horizon, (1) after observing the current price and on-hand inventory (before the current demand is realized), the firm decides how many items from on-hand inventory are allocated to the current period for sale, or equivalently, how many items are preserved for future sale; (2) the demand is realized, the unsatisfied demand is lost or excessive items remain and are further sold in future; (3) the firm pays a holding cost for inventory in each period. We model the firm's optimization problem of initial stock and capacity allocation in a finite/infinite horizon by a Markov decision process (MDP). We focus on the structural properties of the optimal initial stock and capacity allocation policies.

We summarize the research contributions of our work as follows. (1) We show that a save-up-to policy is optimal for the capacity allocation problem, where the save-up-to level is independent of on-hand inventory. This means that the firm should preserve items according to the save-up-to level if possible. It combines features from the base stock policies in inventory control and the booking limit policies in revenue management. (2) We demonstrate the existence of an optimal initial stock. (3) We show that both the optimal save-up-to level and the optimal initial stock are monotone in the retail price, the number of remaining periods, and system parameters (such as holding cost). (4) For the firm, our numerical results provide insights about how to manage efficiently capacity with discretionary sale corresponding to specific market conditions.

The remainder of this paper is organized as follows. In Section 2, we review the literature mainly related to this paper. In Section 3, we present the finite horizon problem and describe its notations and assumptions. We derive the optimal policies and systematically characterize the structural properties of the optimal policies. Section 4 addresses the infinite horizon problem. In Section 5, we perform numerical analysis to confirm some theoretical results derived from the model, and identify market conditions under which dynamic capacity management with discretionary sale is efficient. Section 6 contains concluding remarks.

2. Literature review

This paper is mainly related to three streams of research in the literature. The first one is seat inventory control, where the basic problem is to determine a booking limit for each period or to allocate a capacity for each fare class to maximize the total expected revenue. Littlewood (1972) considers two fare classes for a single-leg flight and studies how to set a booking limit on the number of low-fare tickets available for sale. After Littlewood (1972), there is a large body of literature on seat inventory control for a single-leg flight. We refer interested readers to McGill and van Ryzin (1999) for a comprehensive review. In recent years, the research on seat inventory control has been extended in various ways. Zhang and Cooper (2005) study a simultaneous seat inventory control problem over a set of parallel flights with dynamic customer choice. Netessine and Shumsky (2005) develop a model for seat inventory control under competition and propose a revenue-sharing coordination contract. Ge et al. (2010) study a revenue management problem in the network environment. Kim (2015) examines the impact of customer buying behavior on the optimal airline seat inventory allocation decisions and develops an efficient heuristic algorithm to reduce computation time.

The second one related to this paper is dynamic pricing. Gallego and van Ryzin (1994) is the first paper to consider continuous time dynamic pricing. Their work is extended by Zhao and Zheng (2000) to consider a non-homogeneous Poisson demand and a possible change in consumer reservation price. Bitran and Mondschein (1997) study a dynamic pricing problem of seasonal products with heterogenous consumers. Dai et al. (2005) study pricing competition among multiple retailers with finite capacities. Hu et al. (2010) consider a continuous time dynamic pricing problem in a service supply chain based on customer choice. The above papers consider a product type, while several studies extend them to consider multiple product types. See, for example, Feng and Xiao (2001) and Talluri and van Ryzin (2004).