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Exact accounting of inventory costs in stochastic periodic-review models

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ABSTRACT

Most stochastic periodic-review inventory models calculate inventory costs according to the inventory level at the end of the review cycle. However, in most cases, actual inventory costs accrue at different time-points over the course of the cycle, suggesting that the outputs of these models might be suboptimal. This work proposes a means of overcoming this deficiency by attributing inventory costs to their actual timing. Formulas are presented for the expected profit and for its corresponding optimality equation in various multi-period models that follow complete or partial backlogging, with or without spoilage, stationary or varying inventory costs within cycle, and immediate or postponed revenues. We show that the optimality equation of all these model-variants is a kind of "newsvendor formula" with modified demand distribution and cost parameters. A simple approximation formula and bounds on the optimal order quantity, which are based only on the demand distribution over the entire cycle, are presented and combined together into an improved approximation formula. Numerical examples of a Brownian motion demand process demonstrate the benefit of the proposed approximation in comparison to the simple approximation formula.

1. Introduction

The availability of accurate inventory cost estimates is a major prerequisite for optimizing any inventory policy. Periodic-review systems have been predominantly analyzed under the so-called end-of-cycle[1] accounting scheme (Rudi et al., 2009). According to this scheme, first introduced by Arrow et al. (1951), inventory cost evaluation only uses end-of-cycle inventory information (see, e.g., Guowei et al., 2012). This accounting scheme may be inaccurate since it ignores all inventory variations within the cycle, and its inaccuracy increases as the review period increases. In their seminal paper, Hadley and Whitin (1963) were the first to use the continuous accounting scheme to model the average cost per unit time for all standard periodic-review policies and treated the review period as a decision variable. They provide an exact formulation for Poisson demands (Section 5.3) and for normally distributed demands (Section 5.5) in which the expectation and variance are proportional to the interval length. Since they have established no properties of the respective cost functions in the review period, which are necessary for formal optimization, they proposed the use of exhaustive search to determine optimal policy parameters.

Only recently, specific convexity properties were formally established. Rosling (2002) presents inventory models for a compound renewal process in which the cost function includes duration-dependent inventory costs, as well as per-unit and per-period inventory costs. He shows the cost function is quasi-convex in the inventory position when demand distribution is log-concave, and suggests two approximations of the cost function, but because of the generality of his models, closed-form expressions cannot be obtained for the cost function or for the optimality equation.

Rao (2003) jointly optimized the optimal policy in both the order-up-to level and the review period for the base stock policy (R,T) under the Brownian motion and compound Poisson demand processes. He also established a formal relation between the inventory cost functions prevailing under continuous and end-of-period accounting schemes. Lagodimos et al. (2012a, 2012b) considered the control of a single-echelon inventory installation under the (r,nQ,T) batch ordering policy with stationary random demand and backlogging of unsatisfied demand. They show that whereas the total average cost is not convex, average holding and backorder costs are jointly convex in all policy variables. They also provide computational results for the Brownian motion process, which demonstrates that quantized supplies may cause serious cost increases depending on the supply lot size. Lagodimos et al. (2012c) extended the cost function relation between the two schemes to any periodic-review policy under demands that are stochastically increasing linear in the interval length (e.g., the compound Poisson and the Brownian motion processes). They used it to show that algorithms developed for optimizing cost

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[1] The term "end-of-cycle" is more a precise than "end-of period" in order to cover all periodic review policies and not just the base stock policy where inventory cycle equals review period.

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Notations

β	the discount factor across cycles
t	the index of the cycle (t=1,2,…)
a_t	the inventory level observed at the start of cycle t
Q_t	the ordered quantity for cycle t (a decision variable)
S_t	the order-up-to level for cycle t
$\overrightarrow{S} \equiv (S_1, S_2, \ldots)$	
$D(t)$	the demand during cycle t (a random variable)
$F(x)$	the CDF of $D(t)$
$f(x)$	the PDF of $D(t)$
μ	the expected value of $D(t)$
σ	the standard deviation of $D(t)$
r	the retail price of a unit (when prices are fixed within the cycle)
c	the purchase cost of a unit
γ	the proportion of non-spoiled units
δ	the proportion of backlogged units
H	the holding costs per unit over the entire cycle
P	the shortage costs per unit over the entire cycle
n	the number of epochs in a cycle
k	index (k=1,2,…,n)
$D_k(t)$	the demand during epoch k of cycle t
$\overrightarrow{D}(t) \equiv (D_1(t), D_2(t), \ldots, D_n(t))$	
$F_k(x)$	the CDF of $D_k(t)$
$f_k(x)$	the PDF of $D_k(t)$
μ_k	the expected value of $D_k(t)$
σ_k	the standard deviation of $D_k(t)$
$I_k(t)$	the inventory level at the end of epoch k of cycle t
h_k	the holding cost of a unit during epoch k
p_k	the shortage cost of a unit during epoch k
r_k	the retail price of a unit during epoch k
r_{n+1}	the retail price of a backlogged unit
$g_k(I)$	the inventory cost of I units in inventory at the end of epoch k
$\Pi(S_t, \overrightarrow{D}(t))$	the profit per cycle t
$G(S_t, \overrightarrow{D}(t))$	the total inventory costs during cycle t
$C(S)$	the expected inventory costs per an average cycle under complete backlogging and no spoilage
$C_{pbs}(S)$	the expected inventory costs per an average cycle under partial backlogging and spoilage
$T(\overrightarrow{S})$	the expected total discounted stream of profits
$\overline{\pi}(S)$	the expected profit per average cycle
ω_k	the weight of the inventory costs of epoch k
ω	the critical ratio
X	the cost-weighted demand (a mixture of random variables)
$F_X(x)$	the CDF of X
μ_X	the expected value of X
σ_X	the standard deviation of X
S^*	the optimal order-up-to level
S_N^*	the order-up-to level according to the two-moment normal approximation
$F_{X(j)}(x)$	the CDF of X if all the demand is concentrated in epoch j
S_1^*	the optimal order-up-to level if all the demand is concentrated in the first epoch
S_n^*	the optimal order-up-to level if all the demand is concentrated in the last epoch
S_A^*	the order-up-to level according to the proposed approximation
$F_c(x)$	the CDF of X under the continuous-time cost accounting scheme
S_c^*	the optimal order-up-to level under the continuous-time cost accounting scheme
$\Phi(z)$	the standard normal CDF
$\phi(z)$	the standard normal PDF
$L(z)$	the standard normal loss function
ρ	the ratio between the inventory costs of epoch n and those of previous epochs

under an end-of-period accounting scheme also apply to cost optimization under a continuous accounting scheme. They also provided computational results for Poisson demands, which demonstrated that cost performance of all three standard periodic-review policies ((R,T), (r,nQ,T) and (s,S,T)) is virtually identical when the review period is jointly optimized together with the other policy variables. Shang and Zhou (2010) and Shang et al. (2010), extended Rao (2003) work and sought to optimize an N-echelon serial inventory system under an (r,nQ,T) policy at each echelon. In order to optimize review periods at all echelons, they approximated the continuous accounting scheme by a discrete scheme, considering the review period at each echelon as a product of an elementary unit-period, and modeled the system cost per unit-period. They used a numerical study with Poisson demands to evaluate the sensitivity of the optimal solution and the suggested heuristic solutions to different parameter values.

Rudi et al. (2009), who also followed Rao's work, suggested an approximation formula for the optimal order-up-to level when the cycle length is given, inventory costs accrue continuously and demand is a Brownian motion process with complete backlogging. They suggested an adjusted formula for a model in which costs accrue at multiple evenly spaced points in time, but did not examine its effectiveness. Avinadav (2015), who followed Rudi et al.'s work, found closed-form expressions for the cost function and the optimality equation when demand follows Brownian motion and Poison processes. He suggested an approximation formula to the optimal order-up-to level, which is based on the logistic function, and using numerical study showed it provides better accuracy levels that of Rudi et al. (2009).

According to the literature above, it seems that continuous accounting provides accurate estimates of the true inventory cost, whereas the end-of-cycle accounting scheme just approximates the true cost. However, since this literature assumes full backlogging and no spoilage, it disregards the inventory costs that are associated with the inventory level at the end of the cycle, such as disposal costs of leftover units, as well as shortage costs, which are not time-durable.

While the assumption that the inventory level is a continuous-time variable is reasonable (since demand mostly occurs in continuous time), we claim that inventory costs mostly accrue in specific, discrete time-points. In order to establish our claim, it is important, first, to recognize the components of inventory costs and how they should be accounted for. According to Nahmias (2009, p. 190), the most significant component of holding cost is the opportunity cost of alternative investment. However, even when payments are made immediately at purchase time, they are not immediately invested in order to produce interest. Therefore, the holding cost of a sold unit continues beyond its selling time until the next time point at which accumulated revenues are invested. These time-points are mostly discrete, such as the beginning or ending of a business day, a week or a month. Another type of holding cost that supports the discrete-time accounting scheme is maintenance of inventoried units. Usually, maintenance costs are payments to workers who take care of the leftover units, and such maintenance is carried out at specific time-points (for example, cleaning unsold cars at a dealership at the end of the week or the month). Similarly, shortage costs may accrue in