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A note on a model to evaluate acquisition price and quantity of used products for remanufacturing

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ABSTRACT

Pokharel and Liang [2012. A model to evaluate acquisition price and quantity of used products for remanufacturing. Int. J. Prod. Econ. 138, 170–176] considered a consolidation center that buys used products of different quality levels and sells them together with spare parts to a remanufacturer. The consolidation center's decision problem is to determine the acquisition price to offer for used products and the quantities of spare parts to buy. In this paper, comments on their work are given. It is shown that following Pokharel and Liang's original assumptions, the problem has a trivial solution. We then consider an alternative assumption where supply is uniform and depends on the acquisition price. For this setting, an efficient solution algorithm and numerical examples are provided. In a second model, additional assumptions are relaxed, allowing the consolidation center more flexibility. As expected, this further decreases cost.

1. Introduction

In recent years, remanufacturing has become increasingly popular for ecological as well as economic reasons. The remanufacturing process starts with the reclamation of used products, often called "cores". They are then disassembled, cleaned and inspected. Depending on the quality of the cores, some spare parts may be added and, finally, they are reassembled to some sort of "as good as new" products. In this context, Pokharel and Liang (2012) consider a consolidation center that buys used products from collection centers (which obtained them from customers), combines them with appropriate spare parts corresponding to their quality level and sells both to a remanufacturer. Given a fixed order quantity from the remanufacturer that must be fulfilled and stochastic returns of used products, Pokharel and Liang (2012) propose a model to determine optimal acquisition prices and quantities for the different quality levels. More specifically, they do not decide on the quantity of used products actually bought, but on the planned quantity that equals the number of corresponding spare parts that must be bought in advance before the realization of supply. Moreover, the planned quantities (total number of spare parts) must sum up to the given order size. For reasons of business continuity, everything offered by the collection centers is actually bought.

This paper is organized as follows. In Section 2, comments on the work of Pokharel and Liang (2012) are given. We strictly adhere to Pokharel and Liang's assumptions and identify several shortcomings of their paper. To do so, in Subsection 2.1, we carve out a main assumption that is not explicitly stated in Pokharel and Liang (2012): Despite being a decision variable in their model, the acquisition price does not influence supply. The amount and quality of cores obtained by the consolidation center is independent of the acquisition price. Thus, the only cost-minimizing solution is obviously the lowest acquisition price possible. However, Pokharel and Liang (2012) do not obtain this trivial solution because of a sign error in their analysis of the KKT-conditions, as we show in Subsection 2.2. From our point of view, the existence of the trivial solution renders any further analysis of the problem as given by Pokharel and Liang's assumptions superfluous. For the sake of completeness, we discuss in Subsection 2.3 why the numerical solution procedure developed and used in the remainder of Pokharel and Liang (2012) is highly questionable and does not even get close to the optimal solution in the instances considered.

In Section 3, we present our first model. It is obtained by correcting the key assumption. We now assume that supply depends on the acquisition price offered by the consolidation center. Albeit also other assumptions could be questioned, we think this is the smallest change necessary to arrive at a reasonable problem. Moreover, it ensures analytical tractability. To improve readability, we state the complete problem formulation in Subsection 3.1 and also briefly motivate our choice of the price-dependent supply function. In Subsection 3.2, we derive the KKT

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conditions from the corresponding optimization problem and show that the trivial solution is now no longer necessarily optimal. A solution algorithm is developed in Subsection 3.3 and applied to numerical examples in Subsection 3.4.

In Section 4, a second model is presented. Here, we additionally relax three questionable assumptions from Pokharel and Liang (2012). First, the consolidation center is no longer required to buy all cores offered. Second, we now assume that the quality levels are nested in the sense that the spare parts necessary for a low-quality core are also sufficient for a higher-quality core. Third, the total number of spare parts bought is no longer required to equal the given order size, for example allowing the consolidation center to buy more spare parts to hedge against supply uncertainty. We conclude in Section 5.

2. Comments

2.1. Dependence of used product supply on acquisition price

Pokharel and Liang (2012) never explicitly state how the supply of used products depends on the acquisition price. Their model assumptions only state that "used product supply at quality level n, S_n, is stochastic […]" (PL Assumption 1), "Used product supply quantity at quality level n follows a probability density function $f(S_n)$ with known mean μ_n and standard deviation σ_n" (PL Assumption 2) and that the acquisition price must be in the range between the salvage value r_0 and the per unit underage penalty cost P_0 minus the cost of the corresponding spare parts b_n (PL Assumption 5: $r_0 < p_n < P_0 - b_n$).

Comment. By comparing Eqs. (PL5) and (PL6) we note that the derivative of $S_n p_n$ with respect to p_n is obviously S_n (see (PL6). There is no dependence of the returned quantity S_n on the acquisition price p_n:S_n is not a function of p_n. Given that the acquired quantity of used products does not depend on the acquisition price, one would intuitively expect the lowest possible price to minimize cost. More formally, the objective function stated in Section 2.2 is linear in the prices p_n and increasing. As it is minimized, the smallest possible values are optimal. However, in the two remarks in their Section 3.3, Pokharel and Liang analytically show that the optimal price p_n does not equal the lower or upper bound. They consider this result intuitive because they seem to be not aware of the fact that their model technically does not include any influence of prices on supply.

Moreover, from Eqs. (PL3 and PL4) and later elaborations, it is obvious that the inequality in PL Assumption 5 is not meant in the strict sense, that is, it should read $r_0 \le p_n \le P_0 - b_n$.

2.2. Analysis of the KKT conditions

In the following, we first briefly restate the authors' analytical investigation. Then, comments are given. It is shown that the authors' counterintuitive result is caused mainly by a sign error when applying KKT-conditions.

The starting point for their elaborations is "the cost function, C, for total acquisition [cost] by the consolidation center" (Pokharel and Liang, 2012, Section 3.3)

$$C(p_n, q_n) = \sum_{n=1}^{K} \left[S_n p_n + b_n q_n + P_0 \int_0^{q_n} (q_n - S_n) f(S_n) dS_n - r_0 \int_{q_n}^{\infty} (S_n - q_n) f(S_n) dS_n \right] \quad \text{(PL1)}$$

where the first three elements are costs for acquisition of used products, spare parts and underage quantities, respectively, and the fourth is the salvage value obtained from an overage quantity. To obtain the optimal acquisition price p_n and planned acquisition quantity q_n for each quality level n, (PL1) is minimized subject to the following constraints:

$$\sum_{n=1}^{K} q_n = d \quad \text{(PL2)}$$

$$p_n \ge r_0 \qquad n = 1, \ldots, K \quad \text{(PL3)}$$

$$p_n \le P_0 - b_n \qquad n = 1, \ldots, K \quad \text{(PL4)}$$

These constraints ensure that the sum of the planned acquisition quantities over all quality levels equals the order quantity from the remanufacturer and that the acquisition price is in the range mentioned above. Using (PL1–4), the authors derive the Lagrangian

$$L = \sum_{n=1}^{K} \left[S_n p_n + b_n q_n + P_0 \int_0^{q_n} (q_n - S_n) f(S_n) dS_n - r_0 \int_{q_n}^{\infty} (S_n - q_n) f(S_n) dS_n \right] - \lambda \left(\sum_{n=1}^{K} q_n - d \right) - \alpha_n (r_0 - p_n) - \beta_n (p_n + b_n - P_0) \quad \text{(PL5)}$$

where λ, α_n, and β_n are the Lagrange multipliers associated with the total quantity of used products as well as the lower and upper bounds on the acquisition price, respectively. From the Lagrangian, the following KKT first order conditions are derived using $F(q_n)$ to denote the cumulative probability density function of S_n:

$$\frac{\partial L}{\partial p_n} = S_n + \alpha_n - \beta_n = 0 \quad \text{(PL6)}$$

$$\frac{\partial L}{\partial q_n} = b_n + P_0 F(q_n) + r_0 (1 - F(q_n)) - \lambda = 0 \quad \text{(PL7)}$$

$$\sum_{n=1}^{K} q_n = d, \ \lambda \ge 0, \ \lambda \left(\sum_{n=1}^{K} q_n - d \right) = 0 \quad \text{(PL8)}$$

$$r_0 - p_n \le 0, \ \alpha_n \ge 0, \ \alpha_n (r_0 - p_n) = 0 \quad \text{(PL9)}$$

$$p_n + b_n - P_0 \le 0, \ \beta_n (p_n + b_n - P_0) = 0 \quad \text{(PL10)}$$

Using these conditions, the authors now show in two remarks that optimal prices p_n cannot be equal to the lower bound, but may be equal to the upper bound.

- "If $p_n = r_0$ and $p_n < P_0 - b_n$, then $\alpha_n \ge 0$ (PL9) and $\beta_n = 0$ [… (PL10)]. Otherwise, it will give $S_n \le 0$ or strictly $S_n = 0$ [… (PL6)]". They conclude that offering the lowest price is not optimal.
- "If, $p_n > r_0$ and $p_n = P_0 - b_n$ then $\alpha_n = 0$ [… (PL9)] and $\beta_n \ge 0$ [… by (PL10)]. Then from [… (PL6)], $S_n = \beta_n$, […]." The authors conclude that offering the highest possible price can be optimal. This is described as intuitive because "such a high price can attract the return of more used products".

Comment. Given the stochastic environment and the last two terms, Eq. (PL1) is obviously meant to represent expected cost. Thus, to be formally precise, the first element should be $\mathbf{E}[S_n p_n] = \mu_n p_n$. Correcting for obvious typos such as the omitted last sum, the Lagrangian is given by

$$L(\mathbf{p}, \mathbf{q}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \lambda) = \sum_{n=1}^{K} \left[\mu_n p_n + b_n q_n + P_0 \int_0^{q_n} (q_n - S_n) f(S_n) dS_n - r_0 \int_{q_n}^{\infty} (S_n - q_n) f(S_n) dS_n \right]$$