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Aggregate Return On Investment for investments under uncertainty

Carlo Alberto Magni *

University of Modena and Reggio Emilia, Department of Economics "Marco Biagi", CEFIN – Center for Research in Banking and Finance, viale Berengario 51, 41100 Modena, Italy

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ABSTRACT

This paper deals with capital budgeting decisions under uncertainty. We present an Aggregate Return On Investment (AROI), obtained as the ratio of total (undiscounted) cash flow to total invested capital and show that it is a genuine rate of return which, compared with the risk-adjusted cost of capital, correctly signals wealth creation. For choosing between two mutually exclusive projects, we derive an incremental AROI and an incremental risk-adjusted cost of capital, by means of which two unequal-risk projects can be correctly compared. Iterating the incremental procedure, we show that the AROI approach correctly ranks any bundle of different-risk competing projects. Relations with other criteria such as Modified Internal Rate of Return, average IRR, Cash Multiple, and Profitability Index are provided.

Theoretically, the AROI approach constitutes a link between arbitrage choice theory and corporate investment theory, and shows that explicit discounting is not necessary for measuring economic profitability. Practically, the AROI is a user-friendly, easy-to-compute rate of return derived from the same set of data required by the net present value (NPV). Also, it does not incur the difficulties met by the internal rate of return (IRR): in particular, it is unique and it is based on economically significant capital values (i.e., market-driven values). As such, the AROI significantly expresses the efficiency of the project's invested capital.

1. Introduction

A theoretically correct procedure for investment appraisal and decision under uncertainty is the well-known Net Present Value (NPV). NPV is an absolute measure of worth, expressing the investor's wealth increase in monetary amounts (Peterson and Fabozzi, 2002; Hartman, 2007; Berk and DeMarzo, 2011). Brealey et al. (2011, ch. 34) place the NPV as the first one in the list of the seven most important ideas in finance. In real-life applications, relative measures of worth are also often required. The reason why a relative measure of worth is often searched is that a percentage return is easily understood and felt as an intuitive measure by most investors (Evans and Forbes, 1993). Furthermore, a rate of return supplies information on the efficiency of capital that the NPV cannot supply. For example, consider firm A investing in a one-period investment of 100 at a rate of return of 25%, and let 5% be the cost of capital. The NPV is $125/1.05–100 = 19.05$. Consider firm B investing 1000 in a one-period investment at a rate of return of 7% with the same cost of capital. Then, the NPV is the same: $19.05 = 1070/1.05 - 1000$, but firm A is more efficient, since, for every euro invested, investors earn an active return of $0.25 - 0.05 = 0.2$, whereas firm B's investment generates an active return of only $0.07 - 0.05 = 0.02$ (the poorer efficiency is compensated by a greater investment scale).

Among the relative measures of worth, the most widely used is the internal rate of return (IRR). Both NPV and IRR are massively employed (Remer and Nieto, 1995a, 1995b; Slagmulder et al., 1995; Graham and Harvey, 2001; Sandahl and Sjögren, 2003). In particular, the use of NPV is particularly widespread in industry and engineering (Gallo and Peccati, 1993; Naim, 2006; van der Laan, 2003; Giri and Dohi, 2004; Borgonovo and Peccati, 2004, 2006). The IRR is often employed as well, not only in industry and engineering, but also in real estate and investment performance measurement (Jaffe, 1977; Graham and Harvey, 2001; Geltner, 2003). These two criteria are often used together, and other criteria are also employed such as the profitability index, residual income (e.g., EVA), return on investment, payback period (Remer et al., 1993; Lefley, 1996; Graham and Harvey, 2001; Sandahl and Sjögren, 2003; Lindblom and Sjögren, 2009; Magni, 2009; Hahn and Kuhn, 2012; Pasqual et al., 2013).

Unfortunately, IRR often conflicts with NPV and suffers from many weaknesses, some of them only recently discovered (see Magni, 2013). Among the difficulties, particularly compelling is the fact that IRR is not capable of correctly ranking competing projects. Some scholars advocate the use of an incremental IRR for this kind of problems, but the difficulties of IRR reverberate on incremental IRR: the incremental IRR may not exist or multiple incremental IRRs may exist. Most importantly, the incremental IRR is not applicable when two

* Tel.: +39 059 2056777; fax: +39 059 2056937.
E-mail address: magni@unimo.it
URL: http://morespace.unimore.it/carloalbertomagni/

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(or more) projects have different risks, not even if it exists and is unique: there are two (or more) risk-adjusted costs of capital, one for each project, so it is not clear which one should be compared with the incremental IRR in order to determine the preferred alternative.

In this paper we consider investments under uncertainty and describe a simple, intuitive, metric to capture a project's economic profitability. For this purpose, we build upon Magni (2011) and make use of an Aggregate Return On Investment (AROI), which is a modification of the average internal rate of return (AIRR) introduced in Magni (2010). In particular, resting on arbitrage choice theory, we show that the comparison of AROI and the risk-adjusted cost of capital (COC) signals wealth creation. Contrary to IRR, the AROI exists and is unique; consistently with basic tenets of corporate financial theory, it does not make any assumption on reinvestment of cash flows. This differentiates AROI from the well-known Modified Internal Rate of Return (MIRR): AROI is a project rate of return, MIRR is a rate of return which is an *average* of the project's rate of return and the rate of return of the reinvested cash flows. This also implies that MIRR, as opposed to AROI, is not really unique, because its value depends on the way the project's cash-flow stream is modified. We also show that AROI, while based on undiscounted values, does take time value of money into account, though in a new, indirect way. It is just this feature that enables AROI to give economic significance to some naïve approaches used by real-life practitioners, such as the *cash multiple* and the *undiscounted profitability index*.

We show that an incremental procedure can be applied to AROI in order to correctly rank projects under uncertainty: an incremental AROI is derived, which is compared with an incremental (risk-adjusted) cost of capital (COC), so as to obtain a ranking that is equal to the NPV ranking. Both incremental AROI and incremental COC are weighted averages of the two projects' AROIs and COCs, respectively.

The remainder of the paper is structured as follows. Section 2 introduces a replicating strategy whereby the investor can construct a benchmark asset which replicates the free cash flows of the project. We show that the value of such a benchmark coincides with the capital infused into the project. Section 3 is divided into two subsections: the first one defines the AROI as total return on total capital and shows that it coincides with the ratio of total cash flow on total capital. This implies that the NPV can be framed as an aggregate excess return, namely the product of invested capital and the AROI, net of the risk-adjusted cost of capital. The AROI acceptability criterion is stated. In the second subsection, it is clarified that the AROI approach takes account of the time value of money by incorporating it implicitly. Section 4 deals with choice between unequal-risk mutually exclusive alternatives and project ranking: an incremental procedure is supplied which takes account of the risk of the incremental project. The procedure guarantees that the ranking of projects is equivalent to the ranking via NPV. An illustrative example is presented in the following section. Some concluding remarks end the paper and an Appendix is devoted to describing some relations between the AROI and (i) the Average-Internal-Rate-of-Return approach, (ii) the Modified Profitability Index, (iii) the Modified Internal Rate of Return, (iv) two rules of thumbs, the *Cash Multiple* and the *undiscounted Profitability Index*. The latter metrics, sometimes used by practitioners but considered inappropriate by academics, are resurrected (to some extent) thanks to the AROI approach.

2. The replicating strategy and the benchmark asset

Consider a firm facing a project with initial cost c_0 and let $\tilde{f}_t$ be the (random) free cash flow generated by the project at time $t=1,2,\dots,n$, where n is the maturity date of the last nonzero cash flow (i.e., $\tilde{f}_t=0$ for all $t>n$); let $f_t=E(\tilde{f}_t)$ be its expected value and let $\tilde{s}_n$ the project's residual (scrap) value. The random variables $\tilde{f}_t$, $t=1,2,\dots,n$, $\tilde{s}_n$, and $\tilde{r}$ are assumed to be mutually independent. The expected cash flow vector is $\overrightarrow{f}=(-c_0,f_1,\dots,f_n+s_n)\in\mathbb{R}^{n+1}$ where $s_n=E(\tilde{s_n})$ is the expected residual value. Assume that the capital market is complete and in equilibrium (i.e., no arbitrage exists) and denote as r the risk-adjusted cost of capital (COC), which expresses the minimum acceptable rate of return. The market value of the project is then

$$V_0=\sum_{t=1}^{n} f_t(1+r)^{-t}+s_n(1+r)^{-n}$$

and represents the price the project would have if it were traded. Hence, the project NPV is the difference between value and cost:

$$\mathrm{NPV}_0=V_0-c_0=\sum_{t=1}^{n} f_t(1+r)^{-t}+s_n(1+r)^{-n}-c_0,$$

which measures the investor's wealth increase. More generally, the time-t NPV is denoted as $\mathrm{NPV}_t{:=}\mathrm{NPV}_0(1+r)^t$.

Consider now a shift in perspective: assume an investor that constructs an equal-risk portfolio, denoted as p, which warrants the same payoffs $\tilde{f}_t$ of the project, $t=1,\dots,n$ and ask what the expected terminal value of this portfolio should be in order to get the price of p equal to c_0. Letting s_n^* be such a terminal value, the no-arbitrage principle implies that the following equality must hold:

$$c_0=\sum_{t=1}^{n} f_t(1+r)^{-t}+s_n^*(1+r)^{-n}.$$

This equality shows that p is constructed in such a way that r is its expected rate of return. More precisely, r is the internal rate of return of p and, at the same time, the risk-adjusted COC (i.e., discount rate) for the project. Portfolio p's NPV is zero: in a normal market where no-arbitrage pricing holds, all assets have zero NPV: "The insight that security trading in a normal market is a zero-NPV transactions is a critical block in […] corporate finance. Trading securities in a normal market neither creates nor destroys values." (Berk and DeMarzo, 2011, p. 68.) Being a zero-NPV alternative to the project, portfolio p acts as a *benchmark asset*, with which the project is compared to assess value creation. Note that p replicates the project's free cash flows from time 0 to time n, while leaving a terminal value $\tilde{s}_n^*$ which is, in general, different from $\tilde{s}_n$. Given that the two alternatives share the same free cash flows, acceptance of the project depends on the difference between the expected terminal values, $s_n^*-s_n$: this amount just represents the opportunity cost of investing in the project: the project is worth undertaking if and only if $s_n^*-s_n<0$ (see also Remark 1).

Now, consider that $V_0-\mathrm{NPV}_0$ represents the market value of the project *net of investors' wealth increase*; as we know, this is just equal to c_0 (by definition of Net Present Value), which is the capital invested into the project at time 0. We then generalize this equality and give the following definition of invested capital.

Definition 1. At time $t<n$, the capital c_t invested in a project is given by the difference between the market value of the project and the wealth increase:

$$c_t{:=}V_t-\mathrm{NPV}_t.$$

Let V_t^* be the expected time-t market value of portfolio p. In every period, the following recursive equation holds:

$$V_t^*=V_{t-1}^*\cdot(1+r)-f_t \tag{1}$$

where $V_0^*=c_0$. Armed with the above definition, we show the following result.