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Hybrid heuristic for the inventory location-routing problem with deterministic demand

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ABSTRACT

The Inventory Location-Routing Problem with deterministic demand can be seen as an approach to both optimize a supply chain design and minimize its operational costs. This problem considers that vehicles might deliver products to more than one retailer per route and that inventory management decisions are included for a multi-depot, multi-retailer system with storage capacity over a discrete time planning horizon. The problem is to determine a set of candidate depots to open, the quantities to ship from suppliers to depots and from depots to retailers per period, and the sequence in which retailers are replenished by an homogeneous fleet of vehicles. A mixed-integer linear programming model is proposed to describe the problem and to provide bounds on the solutions. It is strengthened by two sets of valid inequalities with an analysis of their impact. Since the model is not able to solve the targeted instances exactly within a reasonable computation time, a hybrid method, embedding an exact approach within a heuristic scheme, is presented. Its performance is tested over three sets of instances for the inventory location routing, location-routing and inventory-routing problems. Results show important savings achieved when compared to a decomposed approach and the capability of the algorithm to solve the problem.

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1. Introduction

The design of a supply chain is considered as a strategic level decision. It consists of identifying the optimal number of plants to open and their locations so that logistics costs are minimal. On the other hand, the management of a supply chain is usually to tackle tactical and/or operational decisions and it concerns the cooperation between facilities in order to obtain, transform, store and distribute materials, which also entails logistical costs (Melo et al., 2009). Balancing strategic with operational objectives is the challenge.

Most of the facility location models consider distribution to be performed by dedicated routes, i.e., one vehicle visits one client at most (see Gebennini et al., 2009). However, in the case where orders are much smaller than vehicle capacity, this assumption is not longer true. The effects of ignoring routing decisions when locating depots are studied by Shen and Qi (2007) and Salhi and Rand (1989). When vehicles are not performing single-visit tours, locating depots, so that the sum of the distances between depots and retailers is minimized, is not an optimal solution. A more appropriate model is the one depicted in Location-Routing

problems, that propose to optimize location decisions simultaneously with routing decisions. Examples are described by Prins et al. (2007), Belenguer et al. (2011) and a review is presented by Nagy and Salhi (2007). Nevertheless, these papers deal with the single period version or they simplify the multi-period problem by weighting the service to customers to be the same on each period of the horizon. Recently, Prodhon (2011) solved a periodic version, but no inventory decision is managed.

Then, Miranda and Garrido (2009) discuss the impact of ignoring inventory decisions when designing a supply chain. They conclude that the assignment scheme of retailers to depots has a direct impact on depot operation cost because ordering and holding costs might be significantly modified when the aggregated demand varies.

In addition, inventory and routing decisions are strongly interdependent (Bell et al., 1983). Distribution and stock management decisions affect each other for two reasons: First, the set of minimal cost routes is built as a function of the quantities to deliver per period, which are determined by the inventory policies; and second, ordering costs required to design inventory policies include, among others, the transportation cost resulting from the choice of the sequence in which the retailers will be served. The optimal trade-off between inventory and distribution costs is known as the Inventory-Routing Problem (IRP) in Bertazzi et al. (2002), and Andersson et al. (2010).

Designing a supply chain becomes more complex if inventory and routing problems are included in the location decision-making. However, it is essential to balance short-term decisions with longer

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term thinking. As a result, for the Inventory Location-Routing Problem (ILRP), the resulting supply chain design includes an insight into detailed topics in order to decide how to satisfy future demand at minimum cost. Interest arises mainly from two contexts:

(i) When a temporary location is required. It is the case for companies that strategically lease depots and pay rent. Consequently, they are more flexible and might conveniently change locations periodically. It is also the case for humanitarian missions managing disaster relief inventories (Whybark, 2007; Balcik et al., 2010) with limited financial resources through donations. These activities are often performed for a short time. Further, in the field of military logistics, temporary location decisions are often made in order to distribute ammunition and other supplies. In all cases, location costs (e. g., rent) and operational costs (distribution and inventory holding) could have similar orders of magnitude.

(ii) When long-term objectives require a supply chain design allowing different frequencies of replenishment for each retailer and distribution to be performed by vehicles capable of visiting more than one retailer per route. It is the case when assuming single period routing decisions (assuming routing to be the same every period) or dedicated routes (routes visiting a single retailer) are not realistic enough. The large retail sector or pharmaceutical and medical equipment supply are some examples. Again, depot opening costs should be scaled on the modeled horizon to be in balance with the operational costs. Furthermore, even if the future demand is not considered in the long-term, including inventory and routing costs allows incorporating within the location–allocation structure the effects of non-constant distribution activities and the effects of the interactions between inventory and routing decisions. Then, location decisions based on a set of routing scenarios (one per period) will perform outstandingly better on the long-run than one based on a single routing scenario.

Note that these applications suggest that demand might have an unpredictable nature while our model assumes known data. Our contribution is to solve the deterministic version of the problem in order to take the first step before solving a stochastic version with recourse. Even more, we also develop a decision-aid tool for "what-if" analysis. Think that an analyst might be interested in having better estimates of costs given the possibility of restructuring the supply chain under specific future demand assumptions. Few papers simultaneously work on the three problems: depot location, vehicle routing and optimizing inventory policies. Table 1 summarizes a literature review on models and solution methods for the ILRP. Columns **Ret.** and **Dep.** denote if inventory decisions are made either at retailers, at depots, or both.

Most consider a single period routing, location decisions within a discrete set, demand splitting or backlogging not allowed and

stochastic demand. The cost structure to be minimized comprises fixed opening costs for depots, expected holding and stock-out costs, and routing costs. Considering deterministic demand, Ambrosino and Scutellà (2005) propose a linear model for the ILRP and show that for the single period case (LRP), the model implemented in CPLEX 7.0 is not able to find optimal solutions within 25 h for instances with 13 depots and 95 retailers. For stochastic demand, Ma and Davidrajuh (2005) propose an iterative sequential optimization approach where the problem is tackled as a series of sub-problems and never with a global perspective.

In addition, two different characterizations of this problem exist. First, some research papers tackle a LRP integrating in the objective function an EOQ-like component (Wilson model) aiming to minimize the expected inventory management cost at retailers, resulting in a non-linear model. The second approach fixes quantities to be delivered to retailers and optimizes inventory policies at depots instead.

This paper studies the ILRP as the issue of locating depots considering depot fixed opening costs, operational and tactical costs such as routing and stock management cost are included. The mathematical model and some valid inequalities are presented in Section 2. Section 3 describes a hybrid heuristic and a computational study is presented in Section 4. Conclusions are given in Section 5.

2. Problem definition

This paper tackles the design of a two-echelon supply chain considering both strategic and tactical/operational costs. This design comprises the location of the depots supplied by a factory and serving the deterministic demand of retailers, and the assignment of the latter to a depot over a given horizon. Each retailer is assigned to a single depot in the interest of facilitating monitoring and tracing of products. The costs include the depot opening costs, the delivery costs (dedicated routes to depots, non-dedicated to retailers) and the inventory costs at both depots and retailers, including an obsolescence penalty cost (that could be 0 or positive).

Formally, let J be a set of n retailers facing a deterministic non-constant demand $d_{jt}, \forall j \in J, \forall t \in H$, with t a period and $H = \{1, \dots, p\}$ a discrete and finite planning horizon. Also, a set of m candidate depots I is available to replenish retailers. The ILRP is defined on a complete, weighted and directed graph $G = (V, A, C)$. $V = \{J \cup I\}$ is the set of nodes in the graph. C is the cost matrix c_{ij} associated with the traveling cost from node i to node j in the set of arcs A in the network. We consider a homogeneous unlimited fleet of vehicles, thus a set K of r ($r \geq n$) identical vehicles are available. Each node $i \in V$ is associated with a storage capacity W_i. Also, each depot $j \in I$ is associated to an opening cost O_j and ordering cost s_i (dedicated route from the factory or production cost). The vehicle capacity is Q units of product and the fixed cost of using a vehicle at least once in the planning horizon is F. Let B_i be the initial inventory at facility $i \in V$. $H_0 = \{0\} \cup H$ and $H' = H \cup \{p + 1\}$ are horizons including a dummy period used to

Table 1
A classification on combined inventory-location-routing problems and methods.

Authors	Demand	Ret.	Dep.	Model	Solution method
Liu and Lee (2003)	Stochastic	✓		Non-linear	Route first-locate second
Liu and Lin (2005)	Stochastic	✓		Non-linear	Sequential/improv. stage
Ambrosino and Scutellà (2005)	Determ.	✓	✓	Linear	Commercial solver
Ma and Davidrajuh (2005)	Stochastic	✓	✓	Non-linear	Sequential
Shen and Qi (2007)	Stochastic		✓	Non-linear	Branch-and-bound
Javid and Azad (2010)	Stochastic		✓	Non-linear	Tabu search Simulated Annealing
Mete and Zabinsky, 2010	Stochastic		✓	Non-linear	Sequential stochastic programming
Sajjadi and Cheraghi (2011)	Stochastic	✓		Non-linear	Sequential/improv. stage