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Aggregation of exponential smoothing processes with an application to portfolio risk evaluation

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ABSTRACT

In this paper we propose a unified framework to analyse contemporaneous and temporal aggregation of a widely employed class of integrated moving average (IMA) models. We obtain a closed-form representation for the parameters of the contemporaneously and temporally aggregated process as a function of the parameters of the original one. These results are useful due to the close analogy between the integrated GARCH (1,1) model for conditional volatility and the IMA (1,1) model for squared returns, which share the same autocorrelation function. In this framework, we present an application dealing with Value-at-Risk (VaR) prediction at different sampling frequencies for an equally weighted portfolio composed of multiple indices. We apply the aggregation results by inferring the aggregate parameter in the portfolio volatility equation from the estimated vector IMA (1,1) model of squared returns. Empirical results show that VaR predictions delivered using this suggested approach are at least as accurate as those obtained by applying standard univariate methodologies, such as RiskMetrics.

1. Introduction

The aggregation of ARIMA models has been extensively studied in the econometric literature (see, e.g., Amemiya and Wu, 1972; Brewer, 1973; Wei, 1978; Palm and Nijman, 1984; Weiss, 1984; Nijman and Palm, 1990). According to a number of existing results, the parameters and the orders of the aggregated models can be derived by establishing the relationship between the autocovariance structures of the disaggregated and aggregated models. The aggregated parameters are not always easy to determine, especially when dealing with multivariate models, since in most of the cases it is necessary to solve nonlinear systems of equations to recover them, as detailed by Granger and Morris (1976), Stram and Wei (1986), Wei (1990), Drost and Nijman (1993), Marcellino (1999) and Hafner (2008).

This paper focuses on multivariate integrated moving average (IMA) models and obtains closed-form representations for the parameters of the contemporaneously and temporally aggregated model as a function of the parameters of the original one. In particular, assuming that the data generating process is a vector IMA (1,1), our first contribution is to provide closed-form expressions for the autocovariances of the aggregated model. Those allow to derive exact functions linking the unknown aggregate parameters with the data generating process. Although the focus is on a specific class of moving average processes, results are valid for any aggregation frequency and for every level of contemporaneous aggregation.

The IMA (1,1) model is well known in time series analysis since its predictions take the form of the Exponentially Weighted Moving Average (EWMA) recursion, whose forecast function depends on a single smoothing parameter (decay factor) which expresses the weight by which past observations are discounted. In addition, as discussed by Harvey et al. (1994), it is possible to establish a close analogy between the integrated GARCH (1,1) model (Engle and Bollerslev, 1986) for the conditional volatility and the IMA (1,1) model for squared returns, which share the same autocorrelation function. This representation of GARCH models, which is called "ARMA-in-squares", has been widely employed in the context of temporal and contemporaneous aggregation of GARCH models (see Drost and Nijman, 1993; Nijman and Sentana, 1996; Hafner, 2008) and by Francq and Zakoïan (2000), who exploit it to propose a least-squares estimator of weak GARCH models.

The EWMA model is also widely used by practitioners to produce forecasts of volatilities of financial data. It has been

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popularized by RiskMetrics, a risk management methodology for measuring market risk developed by J.P. Morgan based on the Value-at-Risk (VaR) concept. VaR is one of the standard measures used to quantify market risk in the financial industry. VaR is the basis of risk measurement and has a variety of applications, essentially in risk management and for regulatory requirements. For instance, Basel II Capital Accord imposes to financial institutions to meet capital requirements based on VaR estimates at a confidence level of 1% (BCBS, 1996).

VaR evaluation (and prediction) is also the focus of our empirical application, which is an additional contribution of the paper. In the application, we illustrate how the results on temporal and contemporaneous aggregation are useful, for instance, in analysing the problem of volatility prediction and VaR calculation for a portfolio of multiple assets (contemporaneous aggregation) at several sampling frequencies (temporal aggregation). In this paper we are interested in forecasting the volatility of a portfolio composed of several assets, whose individual volatilities are driven by integrated GARCH (1,1)–hereafter IGARCH (1,1)–processes. The portfolio volatility can be described by specifying a univariate model for the portfolio log-returns (portfolio approach) or by contemporaneously aggregating a multivariate model for the system of asset returns contained in the portfolio (asset-level approach). Modeling the joint behavior of the assets in a multivariate fashion can lead to more efficient estimates than those obtained by estimating univariate volatility models and to forecast improvements, mainly because a larger information set is used. In fact, in the framework of contemporaneous aggregation of vector ARMA (VARMA) models, Lütkepohl (1987) has shown that future aggregates can be forecasted more accurately aggregating ex-post the forecasts of the original disaggregate process rather than forecasting the aggregate directly.

At the same time, one can be interested in building portfolio volatility models working with time-aggregated returns and in obtaining time-aggregated VaR measures: Basel II, for example, requires backtesting for an horizon of ten trading days, suggesting the square-root-of-time rule of thumb (see, e.g., Wang et al., 2011). Theory predicts that temporal aggregation entails an information loss: this may affect the efficiency of the estimated parameters and produce a loss of forecasting efficiency (see Wei, 1990). For this reason, econometricians recommend inferring the parameters of the aggregate model from the disaggregate one, incorporating the information content of high frequency data. In Section 4 we provide a Monte Carlo simulation showing the relevant advantages of using our results in empirical analysis.

In this framework, a method is proposed for deriving the MA coefficient and thus the aggregate parameter in the IGARCH (1,1) volatility equation of the portfolio, for different levels of aggregation frequency of the returns. This requires estimating a multivariate IMA (1,1) model by approximate methods, and inferring the aggregate integrated GARCH parameter as a function of the MA matrix coefficients. Our results show that VaR predictions based on the aggregation of the vector IMA (1,1) model are at least as accurate as those delivered by other benchmark univariate methods such as the Student-t EWMA maximum likelihood estimator (MLE), the Student-t GARCH (1,1) MLE and the RiskMetrics approach.

In the GARCH context, temporal aggregation has been studied by Drost and Nijman (1993) and, more recently, by Hafner (2008). Specifically, Drost and Nijman (1993) derived the order conditions and the low frequency parameters for temporally aggregated univariate GARCH models, while Hafner (2008) extended their study to the case of a multivariate GARCH (1,1) model. Our work shares some similarities with this literature but also differs with respect to assumptions and results: indeed, we also derive the parameters of an aggregated GARCH model making use of some known results on aggregation of ARMA models. Yet, our contribution differs from Drost and Nijman (1993) and Hafner (2008) in two ways. First, we propose a framework in which temporal and contemporaneous aggregation can be dealt with simultaneously, while the previous two papers focus on temporal aggregation only.[1] Second, we cover in full details the aggregation of a specific class of VARMA processes, namely the vector integrated moving average model of order one, providing general results that can be employed when assuming the integrated GARCH case for the underlying disaggregate process.

The rest of this paper is structured as follows. In Section 2 we set up the econometric framework, which is based on Lütkepohl (1984a), Lütkepohl (1987). In Section 3 we derive algebraic solutions for the parameters of the contemporaneously aggregated process. In Section 4 we derive algebraic solutions for the parameters of the temporally aggregated process. In Section 5 we first discuss the analogy between the IMA (1,1) and IGARCH (1,1) models, also from an estimation point of view; we then present the empirical application and its results. Section 6 concludes. The proofs are relegated to an appendix.

2. A joint framework for contemporaneous and temporal aggregation of the vector IMA (1,1) model

A framework to deal with contemporaneous and temporal aggregation of vector ARMA (VARMA) type of models has been suggested and formalized by Lütkepohl (1984a), Lütkepohl (1984b), Lütkepohl (1987). In particular, he shows that contemporaneous and temporal aggregation of a VARMA model corresponds to a linear transformation of a "macro process", which constitutes a different representation of the original VARMA. Therefore, the theory of linear transformations of VARMA processes can be applied to derive results on simultaneous temporal and contemporaneous aggregation. Furthermore, it is possible to prove the closeness of VARMA models after aggregation, namely, the fact that the aggregated model is still in the VARMA class and keeps the same structure.

To cover the aggregation issue in full generality, the econometric framework is based on the commonly named "macro processes" above mentioned, introduced by Lütkepohl (1987). As it will become clear in the sequel, this formulation allows us to consider contemporaneous and temporal aggregation *jointly*.

We assume that the data generating process is an Nk–dimensional integrated moving average process of order one, that is, a vector IMA (1,1). In particular, we use a representation similar to the one proposed by Lütkepohl (2007, p. 441):

$$\underset{(Nk\times Nk)}{\begin{pmatrix} \mathbf{I}_N & \mathbf{O}_N & \dots & \mathbf{O}_N \\ -\mathbf{I}_N & \mathbf{I}_N & \dots & \mathbf{O}_N \\ \mathbf{O}_N & -\mathbf{I}_N & \dots & \mathbf{O}_N \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{O}_N & \mathbf{O}_N & \dots & \mathbf{I}_N \end{pmatrix}} \underset{(Nk\times 1)}{\begin{pmatrix} \mathbf{y}_{k(t-1)+1} \\ \mathbf{y}_{k(t-1)+2} \\ \mathbf{y}_{k(t-1)+3} \\ \vdots \\ \mathbf{y}_{k(t-1)+k} \end{pmatrix}}$$

$$= \underset{(Nk\times 1)}{\begin{pmatrix} \mathbf{c} \\ \mathbf{c} \\ \mathbf{c} \\ \vdots \\ \mathbf{c} \end{pmatrix}} + \begin{pmatrix} \mathbf{O}_N & \mathbf{O}_N & \dots & \mathbf{I}_N \\ \mathbf{O}_N & \mathbf{O}_N & \dots & \mathbf{O}_N \\ \mathbf{O}_N & \mathbf{O}_N & \dots & \mathbf{O}_N \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{O}_N & \mathbf{O}_N & \dots & \mathbf{O}_N \end{pmatrix} \begin{pmatrix} \mathbf{y}_{k(t-2)+1} \\ \mathbf{y}_{k(t-2)+2} \\ \mathbf{y}_{k(t-2)+3} \\ \vdots \\ \mathbf{y}_{k(t-2)+k} \end{pmatrix}$$

[1] Similarly, Petkovic and Veredas (2010) studied the impact of both contemporaneous and temporal aggregation in the context of linear static and dynamic models for panel data.