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Testing for unit roots in the possible presence of multiple trend breaks using minimum Dickey–Fuller statistics☆

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ABSTRACT

Trend breaks appear to be prevalent in macroeconomic time series, and unit root tests therefore need to make allowance for these if they are to avoid the serious effects that unmodelled trend breaks have on power. Carrion-i-Silvestre et al. (2009) propose a pre-test-based approach which delivers near asymptotically efficient unit root inference both when breaks do not occur and where multiple breaks occur, provided the break magnitudes are fixed. Unfortunately, however, the fixed magnitude trend break asymptotic theory does not predict well the finite sample power functions of these tests, and power can be very low for the magnitudes of trend breaks typically observed in practice. In response to this problem we propose a unit root test that allows for multiple breaks in trend, obtained by taking the infimum of the sequence (across all candidate break points in a trimmed range) of local GLS detrended augmented Dickey–Fuller-type statistics. We show that this procedure has power that is robust to the magnitude of any trend breaks, thereby retaining good finite sample power in the presence of plausibly-sized breaks. We also demonstrate that, unlike the OLS detrended infimum tests of Zivot and Andrews (1992), these tests display no tendency to spuriously reject in the limit when fixed magnitude trend breaks occur under the unit root null.

1. Introduction

Macroeconomic series appear to often be characterized by broken trend functions; see, *inter alia*, Stock and Watson (1996, 1999, 2005) and Perron and Zhu (2005). In a seminal paper, Perron (1989) shows that failure to account for trend breaks present in the data results in unit root tests with zero power, even asymptotically. Consequently, when testing for a unit root it has become a matter of regular practice to allow for this kind of deterministic structural change. While Perron (1989) initially treated the location of a potential single trend break as known, subsequent approaches have focused on the case where the possible break occurs at an unknown point in the sample; see, *inter alia*, Zivot and Andrews (1992) [ZA], Banerjee et al. (1992), Perron (1997) and Perron and Rodríguez (2003) [PR].

Taking the presence of a linear trend in the data generation process [DGP] as given, among augmented Dickey–Fuller [ADF] style unit root tests it is the Elliott et al. (1996) [ERS] test based on local GLS detrending that is near asymptotically efficient (in the usual sense that the tests lie arbitrarily close to the asymptotic Gaussian local power envelope) when no trend break is present. When a single trend break is *known* to be present, it is now a test based on PR's local GLS detrended ADF statistic which allows for a trend break that is asymptotically efficient. The latter holds provided the break point is known, or is unknown but can be dated endogenously with sufficient precision. However, when a trend break does *not* occur the PR test is not asymptotically efficient, the redundant trend break regressor compromising power. Moreover, the asymptotic critical values for the PR test based on an estimated break point differ markedly according to whether a trend break occurs or not. Precisely, the no break case critical values are substantially left-shifted relative to their break case counterparts. Since the PR test is left-tailed, the no break case critical values need to be employed to avoid over-sizing in the no break case. Consequently, when a break does occur (and can be dated with sufficient precision), the PR test will be under-sized, with an associated loss in power under the alternative, relative to the test based on the non-conservative break critical values. The underlying problem is then essentially one of uncertainty as to whether trend breaks exist in the data or not.

In work which allows for multiple possible breaks in trend, Carrion-i-Silvestre et al. (2009) [CKP] propose a solution to the

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issues raised above. The test procedure outlined in CKP utilizes auxiliary statistics to detect the presence of trend breaks occurring at unknown points in the sample and then uses the outcome of the detection step to indicate whether or not the unit root test employed should include trend breaks in the deterministic specification. CKP use the (multiple) trend break test of Kejriwal and Perron (2010) to select between either one of the standard GLS detrended M tests of Ng and Perron (2001) or the (feasible) likelihood ratio test of ERS (in each case allowing for a constant plus linear trend), and the corresponding test (in each case the multiple breaks generalization of the single break test from PR) allowing for the number of trend breaks identified by the Kejriwal and Perron (2010) test, and with their locations estimated as outlined in Section 5.1 of CKP.

Assuming the trend break magnitudes to be fixed (independent of sample size) CKP show that their method achieves near asymptotically efficient unit root inference in both the no trend break and trend break environments. In the latter case this occurs because their test employs non-conservative critical values by virtue of the convergence of their break point estimators to the unknown break fractions at a sufficiently fast rate. These asymptotic results are, however, somewhat at odds with the finite sample simulations reported in CKP. These show the presence of pronounced "valleys" in the finite sample power functions (when mapped as functions of the break magnitudes) of the tests such that power is initially high for very small breaks, then decreases as the break magnitudes increase, before increasing again. This discrepancy occurs because while the trend break pre-tests used in the CKP procedure are consistent against breaks of fixed magnitude, in finite samples they will not provide perfect discrimination; i.e., some degree of uncertainty will necessarily exist in finite samples as to whether breaks are present or not. The simulation results in CKP suggest, perhaps unsurprisingly, that this problem becomes increasingly pronounced the greater the number of breaks in the series, other things being equal.

For the case of a single break in trend, Harvey et al. (2012) [HLT] show that treating the trend break magnitude to be local to zero (in a Pitman drift sense), rather than fixed, allows the (local) asymptotic distribution theory to very closely approximate this finite sample effect in the CKP test for a single break. This is because the local-to-zero model for the breaks reflects in the asymptotic theory the uncertainty that necessarily exists in finite samples as to whether trend breaks are present in the data or not. Here we show that the finite sample "valleys" problem worsens as the number of trend breaks present increases, other things being equal. Moreover, we show that in the case of multiple trend breaks the pattern of the breaks is also an important factor, in particular whether the parameters on consecutive trend breaks have equal or opposite sign. Our results suggest that the typical trend break magnitudes seen with real macroeconomic data lie well within the valleys region, suggesting that the existing methods may be very poor at discriminating between the unit root null and stochastic stationary alternative in practice.

In response to this problem we propose a practical solution based on a similar approach to that outlined in ZA for the case of a single putative trend break. ZA propose using the infimum of t-ratio-type OLS detrended ADF statistics taken across all candidate break points in a trimmed range. However, it is known that the resulting test can have an asymptotic size of one when a trend break of fixed magnitude occurs under the unit root null; see Vogelsang and Perron (1998) and Harvey et al. (forthcoming). Correspondingly, we show that under a local-to-zero trend break the asymptotic size of the OLS-based infimum test can also exceed the nominal level, approaching unity in some cases. This renders the OLS detrended infimum test too unreliable to be recommended for use in practice. However, for the case of a single possible break in trend, a local GLS detrended implementation of this test is suggested in PR (who also suggest analogous extremum-type tests based on the M and feasible likelihood ratio type tests), although they establish its large sample behaviour only for the case where no trend break occurs. We extend the work of PR by showing that the size problems discussed above for the OLS detrended infimum ADF test do not pertain under local GLS detrending. We further generalize the contribution of PR by developing a local GLS detrended infimum test which allows for multiple possible breaks in trend.

In both the single and multiple trend break cases, we show that these local GLS detrended infimum tests eliminate the aforementioned power valleys, although this necessarily comes at the expense of some loss of power relative to the CKP test when no breaks are present. In a local-to-zero trend break environment and where the putative break fractions are unknown it is not possible to obtain unit root tests which are invariant (even asymptotically) to the break magnitudes, since the unknown break fractions cannot be consistently estimated. However, the results presented in this paper show that both the size and power properties of the infimum tests vary little as a function of the break magnitudes, so that inference based on these tests is essentially unaffected by the break magnitudes. The infimum test also has the practical advantage that it is relatively easy to compute both for a single and multiple putative trend breaks.

The plan of the paper is as follows. In Section 2 we outline our reference multiple (local-to-zero) trend break model. Here we also detail our proposed infimum test (based on local GLS detrended ADF tests) which allows for multiple possible breaks in trend. Section 3 details the large sample distributions of the infimum statistic under local-to-zero trend breaks and for a local-to-unity autoregressive root; asymptotic critical values are given for implementing the infimum test when allowing for a maximum of either one, two or three trend breaks. For the case of a single putative trend break, we also compare the asymptotic local power properties of the infimum test and the CKP test, together with the recently developed adaptive procedures proposed by HLT, the latter being designed to help mitigate the power valley phenomenon when at most one break in trend is permitted. We also demonstrate that the infimum test, when based on local GLS detrended data, does not suffer the problem seen with its OLS counterpart, whose asymptotic size can tend to unity in the presence of a trend break. Section 4 investigates the finite sample behaviour of the procedures, for both the single break case and also the case of two breaks in trend. An empirical illustration using data on primary commodity prices is also provided to highlight the potential usefulness of the proposed infimum tests. Section 5 concludes. Proofs are collected in an Appendix.

In the following '$\lfloor \cdot \rfloor$' denotes the integer part of its argument, '$\Rightarrow$' and '$\xrightarrow{p}$' denote weak convergence and convergence in probability, respectively, '$x := y$' ('$x =: y$') indicates that x is defined by y (y is defined by x), $\circ$ denotes the Hadamard product, and '$1(\cdot)$' denotes the indicator function. Finally, $\mathbb{I}_x := 1(x \neq 0)$ and $\mathbb{I}_x^y := 1(y > x)$.

2. The model and test statistic

We consider a time series $\{y_t\}$ to be generated according to the following DGP,

$$y_t = \mu + \beta t + \boldsymbol{\gamma}' \mathbf{DT}_t(\boldsymbol{\tau}_0) + u_t, \quad t = 1, \ldots, T, \tag{2.1}$$

$$u_t = \rho_T u_{t-1} + \varepsilon_t, \quad t = 2, \ldots, T \tag{2.2}$$

where $\mathbf{DT}_t(\boldsymbol{\tau}_0) := [DT_t(\tau_{0,1}), \ldots, DT_t(\tau_{0,m})]'$, the elements of which, for a generic fraction τ, are the indicator variables, $DT_t(\tau) := 1(t > \lfloor \tau T \rfloor)(t - \lfloor \tau T \rfloor)$. In this model $\boldsymbol{\tau}_0 := [\tau_{0,1}, \ldots, \tau_{0,m}]'$ is the vector of (unknown) putative trend break fractions, with