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Prediction of limit cycle oscillations under uncertainty using a Harmonic Balance method

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ABSTRACT

The Harmonic Balance method is an attractive solution for computing periodic responses and can be an alternative to time domain methods, at a reduced computational cost. The current paper investigates using a Harmonic Balance method for simulating limit cycle oscillations under uncertainty. The Harmonic Balance method is used in conjunction with a non-intrusive polynomial-chaos approach to propagate variability and is validated against Monte Carlo analysis. Results show the potential of the approach for a range of nonlinear dynamical systems, including a full wing configuration exhibiting supercritical and subcritical bifurcations, at a fraction of the cost of performing time domain simulations.

1. Introduction

As design complexity increases, new materials and novel technologies are introduced to new airframes, empirical methods become increasingly difficult to apply, hence a clear need for physics based modelling tools has emerged. Aeroelasticity in particular is a good illustration of this trend and a need for physics based modelling tools has been identified by Noll et al. [1]. Furthermore, predicting the aeroelastic stability of an aircraft should also identify the consequences of variability or uncertainty in model parameters, as discussed by Pettit [2]. Marques et al. demonstrated the significant impact of structural variability on transonic flutter predictions [3,4]. When nonlinearities are present, the amplitude of oscillations can become limited and limit cycle oscillations are observed. This is a problem of considerable practical interest and is well documented for *in-service* aircraft [5,6]. When nonlinearities are de-stabilizing (softening) a subcritical limit-cycle exists. As discussed by Stanford and Beran [7], unstable LCOs can occur below the flutter speed and lead to a hysteretic phenomenon. This type of instability is extremely undesirable because as the flutter speed is reached, the amplitude increases suddenly and significantly, as the speed drops below the flutter point, the LCO will persist.

The presence of nonlinearities, either structural or aerodynamic, poses additional challenges both in terms of complexity and computational resources, these requirements can be exacerbated by the need to quantify the uncertainty due to unknown or variable parameters. Hence, several efforts have been made to address both these issues.

Reduced order modelling is a technique widely utilised to ease the computational burden associated with high-fidelity unsteady simulations, required to capture nonlinear effects. Proper orthogonal decomposition (POD) is commonly used to compress high order data [8,9] and has been implemented in a reliability-based design optimisation framework (RBDO) for aeroelastic problems [10]. Volterra series can be used to model nonlinear responses with historic consideration, hence suitable for transient problems [11]. Recently, recurrent artificial neural networks (ANN), were applied to replicate an input–output relationship and can be used for nonlinear problems, such as LCOs [12], provided the model is sufficiently trained. System identification techniques using describing functions are another alternative to capture unsteady aerodynamic effects in dynamic aeroelastic problems [13,14]. The common limitation of the methods mentioned above is the sacrifice of physical accuracy and parameter space associated with the reduction process, rendering the ROM unreliable outside the limits of the original data. The application of ROMs to uncertainty quantification (UQ) problems is in principle possible, however, the associated increase in the parameter space would require additional computational resources to generate suitable ROMs.

Two promising approaches which do not compromise the underlying physics of the oscillatory behaviour and have been applied to LCOs are: model reduction techniques based on the

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Nomenclature

Roman symbols

a	polynomial coefficients for polynomial chaos expansions
b	semi-chord
$\mathbf{C}$	structural damping matrix
C_L	lift coefficient
C_M	pitching moment coefficient
$\mathbf{E}$	time/frequency domain transformation matrix
$\mathbf{F}$	generalised external force matrix
$\mathbf{J}$	Newton–Raphson system Jacobian
k	linear stiffness coefficient
$\mathbf{K}$	structural linear stiffness matrix
m	structural mass
M	number of independent continuous uncertain parameters
$\mathbf{M}$	structural mass matrix
n_l	Goland wing, number of lag variables in rational approximation
N_H	number of harmonics
$\mathbf{q}$	vector of modal deflections
$\mathbf{q}_{a_i}$	decomposed generalised aerodynamic vectors, $i = 1, \ldots, n_l$
$\mathbf{Q}$	solution vector
r_α	radius of gyration
$\mathbf{R^n}$	Newton–Raphson system residual vector at iteration n
$\mathbf{S^n}$	Newton–Raphson solution vector at iteration n
t	time
V	freestream velocity
w_i	Wagner variables, $i = 1, 2, 3, 4$
x	solution
x_α	distance between mass centre and elastic axis
$\hat{x}_i$	Fourier coefficient of displacement, $i = 1, \ldots, N_H$

Greek symbols

α	aerofoil pitch displacement, angle-of-attack
β	cubic stiffness coefficient
γ	pentic stiffness coefficient
Γ	polynomial chaos basis functions
ϵ_i	constants in Wagner's function, $i = 1, 2$
φ	polynomial chaos random variable
ζ	damping ratio
η	Goland wing, term from rational approximation of generalised aerodynamic forces, $i = 1, \ldots, n_l$
λ	Newton–Raphson relaxation parameter
μ	aerofoil air mass ratio
ξ	aerofoil non-dimensionalised plunge displacement
ρ	air density
$\mathbf{\Phi}$	truncated matrix of eigenvectors
θ	uncertain parameters
ω	fundamental solution frequency
$\bar{\omega}$	aerofoil, frequency ratio, $\bar{\omega} = \omega_\xi / \omega_\alpha$

Subscripts, superscripts and oversets

$()_{a_i}$	decomposed generalised aerodynamic vectors
$()_{nl}$	nonlinear force
$()_{ry}$	rotation about y axis *dof*
$()_{w_i}$	Wagner function representative aerodynamic variables
$()_x$	Duffing oscillator displacement
$()_z$	translation about z axis *dof*
$()_\alpha$	aerofoil pitch *dof*
$()_\xi$	aerofoil plunge *dof*
$()_\phi$	quantity in modal domain
$()_0$	initial condition (when specified)
$()^*$	non-dimensionalised quantity
$\hat{()}$	Fourier coefficient
$\tilde{()}$	equally spaced time domain solution

centre manifold theorem [15] and frequency-based techniques (finite-difference cyclic methods [16], spectral elements in time [17] and Harmonic Balance (HB) methods [6,18,19]). Although the HB method employs global basis functions resulting in system matrices with no sparsity, it offers better temporal convergence than spectral element and cyclic methods [20]. Additionally, convergence problems can occur for the spectral element method during the transition between unstable and stable branches of a subcritical LCO [7], the HB method does not encounter this problem. An overview of different variations of the Harmonic Balance method, such as high-dimensional, incremental, or elliptic HB methods is given by Dimitriadis [21].

The growth in complexity associated with the classical HB method for higher-order nonlinear terms render it inefficient for most practical problems [18]. The High-Dimensional Harmonic Balance (HDHB) method can simplify the treatment of nonlinearities thus making it scalable for more complex problems and can subsequently offer over one order of magnitude reduction in cost [22]. The benefits of the Harmonic Balance approach deteriorate as the number of harmonics retained to solve the problem increase [23].

As for flutter, LCOs are sensitive to parametric variability, which makes the use of stochastic tools attractive to this problem. Beran et al. [24] applied several UQ techniques to an aerofoil LCO problem, where variability was propagated using time domain and cyclic methods. To overcome the difficulties with applying stochastic methods such as Probabilistic Collocation to long time integration problem, Witteveen et al. [25], re-cast LCO time domain results as a function of the resultant frequency. More recently, Le Meitour et al. [26] used a non-intrusive, adaptive formulation of a generalised Polynomial Chaos Expansion (PCE) approach to 2-dimensional LCO problems, the adaptive formulation allowed for the PCE method to give reliable answers in the presence of discontinuities such as supercritical bifurcations.

In this work, an HDHB formulation is exploited to determine the LCO conditions without incurring the costs of time-accurate simulations; the paper then investigates the practicality of using the HDHB approach to propagate parametric variability using a *Non-Intrusive Polynomial Chaos* (NIPC) approach. The paper will first summarise the HDHB formulation, this is followed by the description of the probabilistic approach based on non-intrusive PCE. The impact of variability on the responses amplitudes and motion frequency is assessed and compared against Monte Carlo (MC) results (using both time domain and HDHB methods).

2. Harmonic Balance formulation

The HB formulation used in this work was proposed by Hall et al. [22] for time-periodic flow problems, this methodology was adapted to nonlinear dynamical systems by Liu et al. [27] and is summarised next. Consider a dynamic system with a nonlinearity in stiffness whose behaviour can be described using a simple equation of motion given by:

$$\mathbf{M}\ddot{x} + \mathbf{C}\dot{x} + \mathbf{K}x + \mathbf{K}_{nl}(x) = \mathbf{F}(x, \dot{x}, \ddot{x}, t) \tag{1}$$

Matrices $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ describe the mass, damping and linear stiffness properties of the system respectively and $\mathbf{K}_{nl}(x)$ is the nonlinear component of the stiffness restoring force. The external force, $\mathbf{F}$ can be a function of the motion of the system and/or time. Here