Journal of Economic Dynamics & Control 37 (2013) 1019–1039

Contents lists available at SciVerse ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

A constructive geometrical approach to the uniqueness of Markov stationary equilibrium in stochastic games of intergenerational altruism

Łukasz Balbus [a], Kevin Reffett [b], Łukasz Woźny [c,*]

[a] Faculty of Mathematics, Computer Sciences and Econometrics, University of Zielona Góra, ul. Szafrana 4a, 65-516 Zielona Góra, Poland
[b] Department of Economics, Arizona State University, PO Box 879801, Tempe, AZ 85287-9801, USA
[c] Warsaw School of Economics, Theoretical and Applied Economics Department, Al. Niepodległości 162, 02-554 Warsaw, Poland

ARTICLE INFO

Article history:
Received 3 May 2011
Received in revised form
6 January 2013
Accepted 18 January 2013
Available online 23 January 2013

JEL classification:
C62
C73
D91
040

Keywords:
Stochastic games
Constructive methods
Intergenerational altruism

ABSTRACT

We provide sufficient conditions for existence and uniqueness of a monotone, Lipschitz continuous Markov stationary Nash equilibrium (MSNE) and characterize its associated Stationary Markov equilibrium in a class of intergenerational paternalistic altruism models with stochastic production. Our methods are constructive, and emphasize both order-theoretic and geometrical properties of nonlinear fixed point operators, and relate our results to the construction of globally stable numerical schemes that construct approximate Markov equilibrium in our models. Our results provide a new catalog of tools for the rigorous analysis of MSNE on minimal state spaces for OLG economies with stochastic production and limited commitment.

1. Introduction

Since the pioneering work of Phelps and Pollak (1968), Peleg and Yaari (1973), and Kydland and Prescott (1977, 1980), there has been great interest in studying dynamic economies without commitment. In some cases, the lack of commitment studied is between different generations of private agents, each who seek to develop enduring relations with successor generations that are needed to sustain coordinated action over time. This sort of intergenerational limited commitment problem arises in models of strategic altruistic growth, as well models where agents have preferences consistent with hyperbolic discounting, among others. In other cases, the commitment friction is between public policy agents and decisionmakers in the private economy who are trying to design mutually time-consistent equilibrium public policies. Examples of such situation is Ramsey equilibrium in models of optimal taxation, sustainable sovereign/public debt, and various monetary policy games.

In each of these models, in the end, the question of interest is the existence and characterization of the set of dynamic equilibria. As is well-known, in the presence of intertemporal commitment frictions, there are significant complications in

* Corresponding author. Fax: +48 22 849 53 12.
E-mail address: lukasz.wozny@sgh.waw.pl (Ł. Woźny).

0165-1889/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jedc.2013.01.005

verifying even the existence of subgame perfect equilibrium, let alone characterize the set of such equilibria. For reasons of tractability and numerical computation, researchers have more recently focused on the existence of pure strategy Markov stationary Nash equilibrium (MSNE) defined on ''minimal'' state spaces. Unfortunately, even when the set of subgame perfect equilibrium is nonempty, sufficient conditions that guarantee the existence of MSNE are not so clear. Further, in a great deal of recent applications of stochastic games, one seeks to compute elements of the set of MSNE (e.g., for calibration or estimation exercises). For such problems, new issues arise concerning the mathematical foundations of numerical procedures that are useful for such applied questions (as, in effect, much of this applied work implicitly assumes the existence of *unique* MSNE at each parameter value). Therefore, for the literature emphasizing the quantitative aspects of dynamic equilibria, perhaps the most important such issue is the stability of the set of MSNE in deep parameters. In this paper, we propose a new set of monotone iterative techniques that address all of these questions within the context of a well-studied class of models, namely stochastic overlapping generations models of growth with strategic bequests. In these economies, one assumes no commitment between successor generations, and object of interest is the set of MSNE. An important feature of our approach is to use properties of a stochastic transition structure. Specifically, under the conditions we present, we are able to obtain an Euler equation representation for MSNE that is both necessary and sufficient in all states. We are then able to use this Euler equation to show that *any* MSNE solution under our conditions must necessarily be the solution to a *decreasing*, continuous nonlinear operator that transforms an appropriate space of candidate equilibrium. Having this operator defined, we are able to provide sufficient conditions for MSNE existence in a compact subset of continuous functions. We then provide a sharp characterization of the order structure of the MSNE set (i.e., they are shown to form an ''antichain'').[1]

Next, and perhaps most strikingly, we provide a set of sufficient conditions under which *globally stable* iterative procedures are available for computing unique MSNE.[2] Moreover, our uniqueness result is valid relative to a very broad class of bounded measurable functions. Finally, we present explicit example where our uniqueness conditions do not hold and multiple MSNE exists. In this sense, we show our conditions for global stability are sharp. Our uniqueness result is particularly important as the class of economies for which it holds include parameterizations of stochastic OLG models found often in applied work.

Finally, we address issues related to the numerical approximation of MSNE in our economies. This question is important in applied work as many papers that seek to study dynamic economies without commitment must first numerically approximate MSNE (e.g., to estimate or calibrate the models to data). Therefore, we provide a catalog of theoretical results characterizing the properties of simple approximation schemes (e.g., discretization methods) that can be used to compute MSNE.

The remainder of the paper is organized as follows: Section 2 discusses how our methods and results fit into the existing literature. Section 3 defines the class of models we initially consider. Section 4 provides conditions under which our economies have (pure-strategy) MSNE, and under which the set of MSNE is a singleton. In Section 5, we provide extensions of our results based upon so-called ''mixed monotone'' operators, which allow us to obtain results for the nonseparable utility case. We also describe methods that construct approximate solutions for MSNE that achieve uniform error bounds relative to a simple discretization method (see Section 5.4 for example). Section 6 concludes with a discussion of applicability of our methods to other classes of stochastic games. At the end of the paper, we include an appendix that presents some definitions, a few abstract fixed point theorems that we use in the paper, as well as the proofs of all our results.[3]

2. Related literature

The environment we consider has long history in economics and dates back to the early work of Phelps and Pollak (1968) and Peleg and Yaari (1973).[4] The economy consists of a sequence of identical generations, each living one period, and deriving utility from its own consumption, as well as that of a successor generation. As agents cannot commit to plans, the ''dynastic family'' faces a time-consistency problem. In particular, each current generation has an incentive to deviate from a given sequence of bequests, consume a nonsustainable amount of current bequests, leaving little (or nothing) for subsequent generations.

Within this class of economies we study, conditions are known for the existence of semicontinuous MSNE, and have been established under quite general conditions (e.g. Leininger, 1986; Bernheim and Ray, 1987). An important step forward in characterizing the equilibrium was made in the work of Amir (1996b), where he introduces stochastic convex transitions structures into the game, and is able to establish the existence of MSNE in the space of continuous functions. This result has been further extended by Nowak and coauthors (e.g., see Nowak, 2006 or Balbus and Nowak, 2008). In this latter work, a key innovation was to introduce a class of stochastic transition structures that are assumed to be an

[1] We provide all the requisite definitions later in the paper when the results are presented.

[2] It bears mentioning obtaining sufficient conditions for the existence of *unique* MSNE has been an open question in the literature (e.g., see the discussion in Amir, 1996b).

[3] Apart from the proof of the equilibrium uniqueness result, which is included in the body of the paper.

[4] Versions of our model under perfect commitment have been also studied extensively in the literature, beginning with the important series of papers by Laitner (1979a,b), Loury (1981), and including more recent work of Alvarez (1999) or Laitner (2002) a.o.