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# Context dependent beliefs☆

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ABSTRACT

This paper examines a model where the set of available outcomes from which a decision maker must choose alters his perception of uncertainty. Specifically, this paper proposes a set of axioms such that each menu induces a subjective belief over an objective state space. The decision maker's preferences are dependent on the realization of the state. The resulting representation is analogous to state-dependent expected utility within each menu; the beliefs are menu dependent and the utility index is not. Under the interpretation that a menu acts as an informative signal regarding the true state, the paper examines the behavioral restrictions that coincide with different signal structures: elemental (where each element of a menu is a conditionally independent signal) and partitional (where the induced beliefs form a partition of the state space).



## 1. Introduction

Both intuition and psychological evidence insist that a decision maker's (DM's) preference over alternatives is affected by the environment in which the decision is made (Kahneman and Tversky, 1984; Simonson and Tversky, 1992; Sen, 1993). While there are many external factors that potentially exert influence on the decision making process, this paper examines a model in which the set of alternatives that is *currently* available acts as a frame – a process often differentiated from general framing effects under the moniker *context dependence*. I identify the behavioral conditions for context dependent beliefs, when the DM's subjective assessment of the likelihood of events depends on the available alternatives (the menu) from which he must choose, and consider additional restrictions that correspond to particular subjective information structures.

Context dependence is often associated with notions of bounded rationality or psychological heuristics (Tversky and Simonson, 1993). This paper, however, interprets menu-induced framing as rational, exploring how and when such behavior exists within the subjective expected utility paradigm. If the DM believes the menu itself contains information regarding payoff relevant uncertainty, conditioning his preference on such information is a rational action. Specifically, the model assumes the payoff associated with each alternative is ex-ante uncertain. The DM's utility from consumption depends not only on the chosen outcome, but also on which *state of the world* is realized. The DM, before consumption, is uncertain about the state of the world, but holds a belief (a probability distribution) over the state space; in a given decision problem, the DM maximizes his expected utility according to his belief. When the DM interprets the current selection of alternatives as a signal about the state of the world, his preferences will change across different decision problems in response to his updated beliefs.

Before expounding the finer points of the model, it is worth considering two examples to better illustrate why menu dependent preferences are indeed necessary to explain many decision making scenarios.

**Example 1.A** (*Luce and Raiffa's diner*)**.** On a first date, Katya finds herself in a restaurant at which she has previously never eaten, and which offers chicken ($c$) or steak ($s$). She states her strict preference for chicken ($c \succ s$). However, upon seeing the restaurant also serves frog legs ($f$), she now states her strict preference for steak ($s \succ c \succ f$).

While Katya's preference reversal in the face of a (seemingly) irrelevant alternative cannot be accommodated by the standard theory (as it violates the weak axiom of revealed preference (WARP)), it has a simple, intuitive explanation. She prefers steak when the food is well prepared, but considers chicken more resilient to the inept chef. In the typical restaurant, she believes it is unlikely the food will be well cooked, and hence, has a preference for chicken. However, in the presence of an exotic dish, she deems it is more likely the restaurant employs an expert chef and so, reverses her preference.

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**Example 2.A** (*Sen's date*)**.** After dinner, Katya's date, Mitya, asks whether she would like to end the date and go home ($h$) or go next door and get a drink ($d$). Thinking the date a success, Katya strictly prefers getting a drink ($d \succ h$). However, before she can respond, Mitya offers a third option: the acquisition and consumption of crystal methamphetamine ($m$). Katya now strictly prefers going home ($h \succ d \succ m$).

Here, again, Katya's rather intuitive behavior cannot be explained by standard theory. She understands the offer of methamphetamine as a signal regarding Mitya's character. So, while she would prefer to continue the date as long as it is likely Mitya is reputable, his proposition is sufficient to sway her beliefs away from such a outcome.

These vignettes exemplify two main components of the model. First, it is only the DM's perception of uncertainty that is changing; ex-post tastes are fixed. In other words, if the DM knew with certainty which state of the world was to be realized, he would exhibit a constant preference across menus. Second, the uncertainty is *local*. The realization regarding the quality of the food in one restaurant is not informative about the quality in a different restaurant; that a previous date was virtuous is not evidence that a future date will be.[1]

The first part of this paper axiomatizes a particular type of context-dependence which adheres to these two restrictions. As in Anscombe and Aumann (1963), I examine a DM who ranks *acts* (i.e., functions) from a state space, $S$, into lotteries over consumption, $\Delta(X)$.[2] Naturally, given the motivation, not all of $X$ will always be available. The DM's entertains a family of preferences over acts, indexed by the subset of $X$ that is currently available. Therefore, for each $A \subseteq X$, we see the decision maker's preference, $\succcurlyeq_A$, over $\{f : S \to \Delta(A)\}$. Then, a *menu-induced belief representation* (MBR) is a single utility index, $u : S \times X \to \mathbb{R}$, and a menu-indexed family of beliefs $\{\mu_A\}_{A \subseteq X} \subseteq \Delta(S)$ such that

$$U_A(f) = \mathbb{E}_{\mu_A}\left(\mathbb{E}_{f(s)}(u(s,x))\right) \qquad \text{(MBR)}$$

represents $\succcurlyeq_A$, where $\mathbb{E}_\pi(\varphi)$ denotes the expectation of the random variable $\varphi$ with respect to the distribution $\pi$. Fixing the menu, the DM acts as a subjective expected utility maximizer. The utility index, $u$, is the same across menus. This is the consequence of the main axiom, menu consistency. Menu consistency dictates, conditional on the realization of a particular state, the DM's preference for alternatives is fixed across menus. Therefore, the context effect is entirely characterized by the change in the DM's beliefs regarding the state space. This places clear limits on the type of context effects that can be accommodated by a MBR. Since any change in preference is the consequence of shifting beliefs, context dependence cannot reverse preference over outcomes for which the resolution of the state is payoff irrelevant (note, because the tastes are state-dependent, constant acts are not necessarily certain outcomes). The general model also imposes a continuity condition[3] – if two menus differ only slightly, then so do their associated beliefs.

Since this paper interprets context effects as being entirely driven by informational concerns,[4] it is of interest to understand how the DM uses the context to update his belief. A modeler, who has access to a DM's preferences in a variety of contexts may want to understand what kind of subjective information the DM believes is encoded by each context. Identifying the connection between a context and its induced belief allows a modeler to make counterfactual arguments. For example, understanding that the DM believes frog legs are the mark of a good restaurant (rather than, say, having 3 items on the menu) allows the modeler to predict what the DM would do at a new restaurant. The second part of this paper explores how menus might correspond to the beliefs they induce. In particular, what restrictions indicate that the DM, acting as a Bayesian, holds a prior belief regarding the state space, and interprets each menu as a collection of signals regarding the relative likelihood of each state? What further restrictions allow us to identify the prior and the structure of these signals?

Following the *anything goes* result of Shmaya and Yariv (2016), any MBR can be rationalized by some prior and set of signals. Without imposing any additional structure, Bayesianism imparts no falsifiable restrictions. Thus, a modeler cannot disentangle an irrational DM who chooses a belief at random in each context from a DM who acts rationally according to his prior belief and information encoded by the context. Moreover, and equally damningly, the rationalizing Bayesian model is highly non-unique. In light of this, I consider two more restrictive signal structures and their corresponding behavioral restrictions.

In the first signal structure, an *elemental signal structure*, the DM takes the elements of the menus as signals. Specifically, he assumes that in each state, $s$, element $x$ is included with probability $l(x,s)$ and excluded with $1 - l(x,s)$. Therefore, the collection of included elements (the menu) is the result of a series of conditionally independent random draws. If $l(x,s) > l(x,s')$ then $x$ is more likely to be available in state $s$ than $s'$, so observing $x$ will increase the relative likelihood of state $s$.

**Example 2.B** (*Sen's date, revisited*)**.** Let $S = \{r, d\}$ indicate reputable and depraved characters, respectively. Katya's has MBR preferences and the following utility index:

$$\begin{array}{lll} u(r,h) = 1 & u(r,d) = 5 & u(r,m) = -10 \\ u(d,h) = 1 & u(d,d) = -5 & u(d,m) = -10. \end{array}$$

She initially believes $\mu(r) = \frac{9}{10}$ and $\mu(d) = \frac{1}{10}$. She also believes that, while all dates will offer going home and getting a drink, depraved characters offer meth with probability $\frac{1}{10}$, with reputable characters with only probability $\frac{1}{100}$.

After updating upon seeing the menu $\{h, d\}$, she holds the beliefs $\mu(r) = \frac{891}{981}$ and $\mu(d) = \frac{90}{981}$; her preference is given by $U_{\{h,d\}}(d) = \frac{5(801)}{981} > 1 = U_{\{h,d\}}(h)$. After the menu $\{h, d, m\}$, she holds the beliefs $\mu(r) = \frac{9}{19}$ and $\mu(d) = \frac{10}{19}$; her preference is given by $U_{\{h,d,m\}}(h) = 1 > \frac{-5}{19} = U_{\{h,d,m\}}(d)$.

I show that this behavior is captured axiomatically by the restriction that the same element, included in two different menus, must have the same proportional effect of beliefs. Moreover, given that a DM entertains an elemental signal structure, the effect of each element on relative likelihoods can be identified uniquely. Next, I consider a *partitional signal structure* (a special case of an elemental signal structure). Here, the DM entertains a partition of the state space and each menu indicates a particular event of the partition has obtained. In other words, the DM believes each menu can only occur in a particular subset of the state space.

**Example 1.B** (*Luce and Raiffa's diner, revisited*)**.** Let $S = \{h, m, l\}$ indicate high and medium and low quality food, respectively. Katya's has MBR preferences and the following utility index:

$$\begin{array}{lll} u(h,c) = 12 & u(h,s) = 16 & u(h,f) = 6 \\ u(m,c) = 9 & u(m,s) = 8 & u(m,f) = 5 \\ u(l,c) = 7 & u(l,s) = 4 & u(l,f) = 3. \end{array}$$

[1] Of course, one could tell a different story where there is a dynamic component by which the DM learns about the likelihood of states from experience. This is well outside of the current model.

[2] For a set $Y$, $\Delta(Y)$ is the set of distributions thereover.

[3] This is a vacuous assumption when $X$ is a discrete space.

[4] In contrast to, for example, (Kalai et al., 2002) in which the change in the rationalizing preference may be the result of changing tastes.