Mathematical Social Sciences ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/mss

Advances in the attraction model for inter-group relations

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HIGHLIGHTS

- Homophily is generated by a mechanism of attraction towards similar others.
- Salience is a function of well-known measures of agreement and dissimilarity.
- The attraction mechanism implies quasi-independence.

ARTICLE INFO

Article history:
Received 13 October 2016
Received in revised form 10 March 2017
Accepted 20 April 2017
Available online xxxx

Keywords:
Homophily
Salience
Intra- vs. inter-group ties
Attraction to similar others
Biased net theory

ABSTRACT

The present paper discusses a *model of attraction towards similar others* as a theoretical account of the problem of homophily in social relations. The focus of the paper is on investigating the properties of the model. More specifically, upon presentation of the logic behind the model, the discussion goes on to investigate the effects of social structural conditions (i.e., margins of an association matrix) on the model parameters — especially those referring to the strength of the homophily bias. This investigation leads to a reformulation of the problem of *salience of a characteristic for association*, defined, after Blau (1977) and Skvoretz (1983), in terms of the difference between the actual frequency of intra-group ties and the frequency expected under randomness. The attraction model is then compared with the log-linear model of quasi-independence. The objective of this comparisons is a precise specification of the attraction model's explanatory scope. The paper ends with an illustrative application of the model to GSS data on confiding relations along ethnicity and religious affiliation.

1. Introduction

Homophily, or the tendency for individuals to associate with others like themselves, is a well established feature of social networks (Lazarsfeld and Merton, 1954; McPherson et al., 2001). Through decades of research, this feature has been documented across types of social relations (e.g. friendship, marriage, talking about important matters, asking for advice, visiting at home, to give few examples) and across characteristics (e.g., ethnicity, nationality, political preference, religious affiliation, gender, age, education, occupation, and so on). Homophily means that intra-group ties, or ties connecting members of the same category, are *statistically over-represented* in the sense that they occur more frequently than expected by chance (Blau, 1994; Fararo and Skvoretz, 1989; Smith et al., 2014). While much of the research on homophily so far has been descriptive, aiming to establish differences, if any, in the levels of homophily across categories, societies, or historical times, considerable effort has been invested in developing theoretical mechanisms that can explain it.

For instance, Wimmer and Lewis (2010) discuss various types of tie-formation processes and how they can generate high levels of homophily in conjunction with properties of a social structure of a population. To illustrate, simply having a 'taste' for similar others as associates may not be enough for an intra-group tie to be formed if the similar others are not available for the focal actor, the availability depending on how the population is distributed over categories of a characteristic and the extent to which the categories are separated, physically or socially — a major theme in Blau's macrostructural theory of social integration (Blau, 1977, 1994). Similarly, repeated interactions between the same individuals who are engaged in some joint activity often result in a tie between these individuals (Feld, 1982). If the population is unevenly distributed across these activities, so that members of different categories end up being engaged in different types of activities (e.g., occupational or residential segregation), then the propinquity mechanism can lead to homophily (Feld, 1982; Feld and Grofman, 2009; McPherson and Smith-Lovin, 1987; Schelling, 1971; Schwartz, 1990).

My focus in the present paper is on a specific *theoretical model* of a tie-formation process, namely, the model of *attraction towards similar others*, developed by John Skvoretz (1991, 2013). In

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http://dx.doi.org/10.1016/j.mathsocsci.2017.04.006
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that model, the probability that a tie linking two individuals is between members of the same category depends on three types of parameters, namely, (a) the strength of the taste for similar others as associates, (b) the probability with which members of the given category are selected at random, given the absence of the preference, and (c) the probability with which members of that category initiate social ties . These parameters can be said to correspond to the tie-formation processes discussed by Wimmer and Lewis (2010): in-group preference, availability, and sociality, respectively, where the latter refers to the ability of developing large numbers of ties with others. Admittedly, the attraction model is rather simple in comparison with statistical models for social networks, such as exponential random graph models (Koehly et al., 2004; Robins et al., 2001, 2007) or SIENA (Snijders, 2005). Unlike the latter, the attraction model is not dynamic and, unlike the former, it cannot be used to show, for example, how homophily can be amplified by reciprocity and triadic closure (Wimmer and Lewis, 2010). On the other hand, the attraction model is rooted in a broader research program on tie-generating processes, namely, biased net theory (Rapoport, 1979; Skvoretz et al., 2004). In that program, the effects of various network processes – in-group preference, triadic closure, or reciprocity – are modelled mathematically in the same consistent manner, which opens up possibilities for extending the attraction model to include these others processes in one consistent formulation.[1]

Previous work on the attraction model dealt primarily with explicating its underlying logic, especially in contrast with a theoretical model of *repulsion from dissimilar others* (Skvoretz, 2013) and the log-linear model of quasi-independence (Skvoretz, 1991). In the present paper, I am taking a closer look at *formal properties* of the attraction model. It applies to data in the form a square contingency table cross-classifying social categories of associates in the given type of social relation. Margins of such tables indicate how many ties originate in each category and how many ties each category receives. Obviously, the margins impose bounds on the values of the model's parameters and it is important that these bounds be specified explicitly. While it may seem like a minor issue, it is instrumental in the second contribution of the present paper, namely, *generalisation of the concept of salience* of a characteristic. The concept plays an important role in Blau's theory (Blau, 1977, 1994; Blau and Schwartz, 1984; Skvoretz, 1983); it is defined in terms of the difference between observed frequency of intra-group ties and the frequency expected under the assumption of stochastic independence between the characteristic and social association. Earlier work by Skvoretz (1983) provides a formal treatment of the concept of salience, but only for populations which meet fairly restrictive criteria of *uniform biased nets* (Fararo, 1981; Skvoretz, 1983, 1991), namely, (a) the requirement that the strength of the homophily bias is the same in all categories of the characteristic and (b) the stipulation that social categories initiate and receive ties of a given kind with probabilities equal to their population proportions . These criteria imply a symmetric association matrix. In the present paper, I use the result concerning the effect of margins mentioned above and go on to show how salience can be expressed as a function of two well-known quantities: (a) Cohen's coefficient κ for measuring agreement and (b) the index of dissimilarity Δ . Also, under the present reformulation, salience can be compared across populations, characteristics, or types of relation. Note that the original definition can be recovered from the present one as a special case.

Another contribution of the present paper consists in investigating statistical properties of the matrix implied by the attraction model. More specifically, I follow Karpiński and Skvoretz (2015) who studied the structure of association (in the statistical sense) in a set of matrices derived from the repulsion model mentioned above using *local odds ratios* to demonstrate correspondence between that model and log-linear models for square contingency tables. Similarly, in the present paper I calculate a set of local odds ratios for a table of tie frequencies resulting from the attraction model. The table turns out to satisfy a property of *quasi-independence* (Goodman, 2007) which can be represented in the form of a log-linear model (Agresti, 2002 chap. 10). This result provides a link between the parameters of the attraction model and the parameters in the log-linear model of quasi-independence. Implications of this finding are discussed in detail at the end of the paper.

The next section presents an overview of the attraction model. Then, I turn to discussing the effect of marginal distributions on the strength of the homophily bias, focusing on its upper bound. I subsequently use this discussion to reformulate the concept of salience, as mentioned above, and then I proceed to investigate the properties of the attraction model using odds ratios and log-linear models. Finally, I illustrate the use of the attraction model by fitting it to data on confiding relations with respect to ethnic background and religious affiliation.

2. Attraction model

As mentioned in the introduction, the attraction model for inter-group relations has its roots in the biased net theory of social structure (Fararo and Sunshine, 1964; Fararo, 1981; Rapoport, 1963, 1979; Skvoretz, 1985, 1990; Skvoretz et al., 2004), which models social networks as resulting from both random and non-random elements, where the latter pertain to systematic relational tendencies of different types, such as *the reciprocity bias*, or the idea that a line from y to x is more likely than chance if there is already a line from x to y, or *the transitivity bias*, or the idea a line from x to y is more likely than chance if they are both connected to a common 'acquaintance' z (Rapoport, 1963, 1979).

Using the former definitions as guidelines, Fararo (1981) defined *the inbreeding bias* as the idea that x and y are more likely than chance to share an attribute (e.g., belong to the same category) if they are connected by a tie. This definition provided the foundation for the effort to formalise Blau's influential macrostructural theory of inter-group relations (Blau, 1977, 1994), in which networks of social relations are viewed as a joint outcome of *individual tendencies* to associate with those of the same social background (i.e., homophily) and *opportunities to associate* provided by social structure, which is defined in terms of the distribution of members of a population along various dimensions, such as gender, ethnicity, religious affiliation, social class, and the like.

The logic behind the attraction model can be explicated as follows. Let G be a set of N social actors and let R be a set of ties, i.e. pairs of members of G that satisfy a social relation of a given kind. Formally, R is a binary, irreflexive and symmetric social relation that can be represented as a square adjacency matrix X of order $N \times N$ such that

$$x_{po} = \begin{cases} 1 \iff pRo \\ 0 \iff \neg pRo. \end{cases} \quad (1)$$

Also, let C be a characteristic that divides G into a set of m mutually exclusive groups, or categories, $G_1, G_2, \ldots, G_m$ with $N_1, N_2, \ldots, N_m$

[1] That both inbreeding bias, the triadic closure, and reciprocity bias can be modelled within the same biased-net framework was used by Fararo and Skvoretz (1987) in their effort to synthesise Blau's macrostructural theory of Granovetter's (1973) theory of the strength of weak ties. Both are theories of social integration, although the integration is viewed somewhat differently in each of them. For Blau, it is defined in terms of the frequency of occurrence of inter-group ties. For Granovetter, it refers to the connectedness of the network, or the proportion of the group that can be reached through relational paths from a randomly selected 'seed'. Fararo and Skvoretz's (1987) proposition derives a measure of connectedness that depends on both the triadic closure and the strength of the homophily bias. The task ahead is to define the probability of a tie from actor x to y as a function of these two parameters as a way of extending the attraction model.

Please cite this article in press as: Karpiński, Z., Advances in the attraction model for inter-group relations. Mathematical Social Sciences (2017), http://dx.doi.org/10.1016/j.mathsocsci.2017.04.006.