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Noise-induced shifts in the population model with a weak Allee effect

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HIGHLIGHTS

- Noise-induced shifts in Truscott–Brindley system are considered.
- Stochastic excitability and Canard explosion are studied.
- Stochastic sensitivity function technique is used for parametric analysis.

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ABSTRACT

We consider the Truscott–Brindley system of interacting phyto- and zooplankton populations with a weak Allee effect. We add a random noise to the parameter of the prey carrying capacity, and study how the noise affects the dynamic behavior of this nonlinear prey–predator model. Phenomena of the stochastic excitement and noise-induced shifts in zones of the Andronov–Hopf bifurcation and Canard explosion are analyzed on the base of the direct numerical simulation and stochastic sensitivity functions technique. A relationship of these phenomena with transitions between order and chaos is discussed.

1. Introduction

In nonlinear stochastic systems, even a weak noise can generate unexpected phenomena that have no analogues in the deterministic case [1,2]. Various stochastic regimes such as a stochastic resonance [3,4], noise-induced transitions [5,6], stochastic bifurcations [7–9], noise-induced chaos and order [10,11], and stochastic excitability [12–14] still attract attention of many researchers from the different domains of science. Similar noise-induced phenomena have been found in nonlinear biological systems. In this actively developed research direction, a study of stochastic effects in neuronal dynamics dominates [15–19]. However, many interesting stochastic phenomena are observed also in other domains of life science, in population dynamics in particular [20–22]. A wide variety of nonlinear functional responses in models of interacting populations [23,24], along with the inevitable noise, can generate a noise-induced extinction, multimodality, ecological shifts, and so on (see e.g. [25–30]).

Commonly, these stochastic phenomena are associated with the multistability of the initial deterministic models. Indeed, in dynamic systems with coexistent attractors, random disturbances can result in the noise-induced hopping between basins of attraction, and consequently, cause new stochastic regimes [31,32]. But even in unimodal systems with a single attractor, random noise can generate a rich variety of unexpected stochastic phenomena [13]. In population dynamics, the prey–predator plankton model introduced by Truscott and Brindley is a typical example of such unimodal but excitable system [33–35].

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In present paper, we consider this Truscott–Brindley model with the embedded weak Allee effect (TBWA). The Allee effect is defined as a decrease of the growth rate (weak Allee effect) or extinction (strong Allee effect) at low population size [36–39]. Noise-induced phenomena in stochastic population models with the Allee effect were studied in [40–43].

Our aim is to show and study, both numerically and analytically, how noise drastically deforms dynamics of TBWA model: shifts the probabilistic distributions, and causes stochastic *P*- and *D*-bifurcations with order-chaos transitions. Our theoretical approach is based on the stochastic sensitivity functions (SSF) technique [44] (a short mathematical background can be found in [45]). This constructive technique allows us to approximate probabilistic distributions of random states near the deterministic attractors (equilibria and limit cycles). Note that even in the two-dimensional case, it is difficult to find such probabilistic distributions directly from the Fokker–Planck–Kolmogorov equation [46]. Mathematically, the stochastic sensitivity function is an asymptotics of the quasipotential [47] for small deviations from the attractor.

Our paper is organized as follows.

In Section 2, we introduce the TBWA model and give a brief review of its dynamic regimes in dependence of the parameter a of the prey–predator relationship. Here, Andronov–Hopf bifurcation is accompanied by the Canard cycles explosion.

Section 2 is devoted to the analysis of noise-induced phenomena in the stochastic variant of this population model with the randomly forced parameter of carrying capacity.

The stochastic sensitivity of attractors (equilibria and cycles) of TBWA model is studied in Section 3.1. Here, a supersensitive Canard cycle is found.

In Section 3.2, noise-induced excitability of the equilibria near Andronov–Hopf bifurcation is studied. A probabilistic mechanism of the transition from small- to large-amplitude stochastic oscillations is analyzed with the help of the confidence ellipses.

Section 3.3 is devoted to the study of the stochastic generation of large-amplitude oscillations in a zone of the Canard explosion.

Noise-induced shifts of probabilistic distributions are demonstrated and studied in Section 3.4. In Section 3.5, we show how these stochastic phenomena are accompanied by "order-chaos" and "chaos-order" transitions.

2. Deterministic Truscott–Brindley model with weak Allee effect

Consider the deterministic system

$$\begin{aligned} \delta\dot{x} &= rx^{\alpha}(k-x) - \frac{a^2x^2}{1+b^2x^2}y \\ \dot{y} &= \frac{a^2x^2}{1+b^2x^2}y - my. \end{aligned} \tag{1}$$

This system models an interaction of the prey and predator. Here, x and y are densities of the prey and predator, correspondingly, r is an intrinsic rate of the prey, k is its carrying capacity, m is a rate of the natural mortality of the predator. The interaction of the prey and predator is defined by the Holling-type III grazing with maximum rate a^2/b^2. The small parameter δ reflects a high sensitivity and fast response of the prey population to environmental changes [34].

For $\alpha = 1$, this system is a well-known deterministic Truscott–Brindley model [33] which describes an interaction of the phytoplankton (prey) and zooplankton (predator).

In current paper, we study the Truscott–Brindley model with the embedded weak Allee effect. A population exhibiting a weak Allee effect possesses a reduced growth rate at lower population density [39]. The parameter α regulates a strength of this reducing. In our paper, we consider the system (1) with $\alpha = 2$. For any parameters, the system (1) has equilibria $(0, 0)$ and $(k, 0)$. The equilibrium $(0, 0)$ is unstable, and $(k, 0)$ is stable for $a < a_1 = \sqrt{m(b^2 + 1/k^2)}$. For $a > a_1$, the equilibrium $(k, 0)$ is unstable, and the system (1) possesses a nontrivial equilibrium $M(\bar{x}, \bar{y})$ with positive coordinates

$$\bar{x} = \sqrt{\frac{m}{a^2 - b^2m}}, \quad \bar{y} = \frac{r}{m}\bar{x}^2(k - \bar{x}).$$

In what follows, we fix a set of parameters: $\delta = 0.1, r = k = m = 1, \ b = 7$, and study dynamics of the population system (1) under the variation of the parameter a. For these parameters, $a_1 = \sqrt{50} = 7.07107$.

At the point $a_2 = 7.1639$, the system (1) undergoes Andronov–Hopf bifurcation, the equilibrium M becomes unstable, and a stable limit cycle appears.

In Fig. 1, the extreme values of the variables x and y are plotted for considered attractors (equilibria and cycles). Typical phase portraits are shown in Fig. 2 for different values of the parameter a. For $a = 7$, all trajectories starting from the nontrivial initial data tend to the equilibrium $(k, 0)$ (see Fig. 2(a)). From biological point of view, this means that the predator is extinct, and the density of prey is stabilized to the value $\bar{x} = 1$.

For $a = 7.16$, all trajectories tend to the nontrivial equilibrium M (see Fig. 2(b)), so the populations of prey and predator coexist in the equilibrium mode.

In Fig. 2(c), limit cycles which describe oscillatory regimes of the coexistence of prey and predator are shown for three values of the parameter a. As one can see, a transition zone from equilibria to large-amplitude cycles is very narrow. Such sharp increase of the amplitude of the cycle is known as a Canard explosion. In Fig. 2(c), it is shown how small changes in the parameter a lead to the appearance of the large-amplitude oscillations.