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A Fractional Model with Parallel Fractional Maxwell Elements for Amorphous Thermoplastics

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Abstract

We develop a fractional model to describe the thermomechanical behavior of amorphous thermoplastics. The fractional model is composed of two parallel fractional Maxwell elements. The first fractional Maxwell model is used to describe the glass transition, while the second component is aimed at describing the viscous flow. We further derive the analytical solutions for the stress relaxation modulus and complex modulus through Laplace transform. We then demonstrate the model is able to describe the master curves of the stress relaxation modulus, storage modulus and loss modulus, which all show two distinct transition regions. The obtained parameters show that the modulus of the two fractional Maxwell elements differs in 2-3 orders of magnitude, while the relaxation time differs in 7-9 orders of magnitude. Finally, we apply the model to describe the stress response of constant strain rate tests. The model, together with the parameters obtained from fitting the master curve of stress relaxation modulus, can accurately predict the temperature and strain rate dependent stress response.

Keywords: Fractional calculus, Viscoelastic, Thermoplastic, Mittag-Leffler

1. Introduction

With the ability to describe the memory and nonlocal behaviors, fractional calculus has been successfully applied in various areas, including diffusion [1–6], chaos [7], chemical reactions [8], dynamics [9], nonlocal elasticity [10], etc. The fractional models are typically developed by replacing the integer order differential equations with the non-integer order differential equations. Compared with the conventional integer order models, fractional order models are able to accurately describe the complex behaviors of materials and systems with fewer parameters [11, 12]. This advantage has also been extensively demonstrated by applying fractional calculus to describe the viscoelastic behaviors of materials [13]. The pioneering work of Koeller [14] extended the rheological Kelvin-Voigt and Maxwell models to the corresponding fractional models by introducing fractional dashpot. Schiessel et al. [15] obtained the analytical solutions for the stress relaxation modulus, complex modulus and creep compliance of the fractional Maxwell, Kelvin-Voigt, Zener and Poynting-Thomson models. Bagley and Torvik [16] demonstrated a fractional viscoelastic model is consistent with the second law of thermodynamics.

In recent years, researchers have used fractional derivative models to explain the complex viscoelastic behaviors of various material systems [17–27]. Katicha et al. [28] used the generalized fractional Maxwell model to predict the storage modulus and loss modulus of different mixes of asphalt concrete. Yin et al. [29] demonstrated the fractional order constitutive models can accurately represent the creep and relaxation behaviors of geomaterials in triaxial tests. Fractional models were also employed to represent the linear viscoelastic behaviors of biomaterials, such as cells [30] and human tissues [31]. Xu and Chen [32] developed the fractional model to describe the creep behavior of Hami

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