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An extended continuum model considering optimal velocity change with memory and numerical tests

Zhai Qingtao [a], Ge Hongxia [a,b,c], Cheng Rongjun [a,b,c,*]

[a] Faculty of Maritime and Transportation, Ningbo University, Ningbo 315211, China
[b] Jiangsu Province Collaborative Innovation Center for Modern Urban Traffic Technologies, Nanjing 210096, China
[c] National Traffic Management Engineering and Technology Research Centre, Ningbo University Sub-centre, Ningbo 315211, China

HIGHLIGHTS

- An extended macro model considering optimal velocity changes with memory is proposed.
- The new model's stability condition and KdV-Burgers equation are deduced.
- The numerical simulation and energy consumption is carried out.

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ABSTRACT

In this paper, an extended continuum model of traffic flow is proposed with the consideration of optimal velocity changes with memory. The new model's stability condition and KdV–Burgers equation considering the optimal velocities change with memory are deduced through linear stability theory and nonlinear analysis, respectively. Numerical simulation is carried out to study the extended continuum model, which explores how optimal velocity changes with memory affected velocity, density and energy consumption. Numerical results show that when considering the effects of optimal velocity changes with memory, the traffic jams can be suppressed efficiently. Both the memory step and sensitivity parameters of optimal velocity changes with memory will enhance the stability of traffic flow efficiently. Furthermore, numerical results demonstrates that the effect of optimal velocity changes with memory can avoid the disadvantage of historical information, which increases the stability of traffic flow on road, and so it improve the traffic flow stability and minimize cars' energy consumptions.

1. Introduction

Up to date, kinds of traffic problems such as traffic congestion, air pollution, energy consumption, traffic noises, traffic accidents, greenhouse effect, parking problem and global warming, are getting steadily worse with great increase of traffic flux [1–13]. Therefore, more and more scholars have been devoted to the study of traffic flow. Until now, a variety of traffic models have been developed from different viewpoints, which can figure out the complicated constitution behind the traffic congestion phenomenon. In general, existing traffic flow models can be divided into four categories, the car-following models [14–26], the cellular automaton models [27–30], the gas kinetic models [31–33], and the hydrodynamic models [34–36]. The optimal velocity model (OVM) is one of the most famous car-following models, proposed by Bando in

* Corresponding author at: Faculty of Maritime and Transportation, Ningbo University, Ningbo 315211, China.
E-mail address: chengrongjun@nbu.edu.cn (C. Rongjun).

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1995 [37]. Inspired by OVM, many new car-following models have been developed taking other traffic factors into account [14–26]. With the consideration of full velocity difference, Jiang proposed a full velocity difference model (FVDM) [38], which conquers the drawbacks of OVM, i.e. unrealistic deceleration and high acceleration rate. Tang put forwarded some traffic models account for driver's bounded rationality [39–42].

It is well known that driver's memory is indispensable and important in driving. The car-following models which considering the effect of memory shown that the historic information will enhance the stability of traffic flow [43–46]. The velocity and headway changes with memory have been studied in traffic flow models by some scholars [47–50]. Numerical simulations demonstrate that driver's memory has significant positive effect on the traffic behaviors and stability. In the actual traffic environment, the optimal velocity changes with memory can predict the change of acceleration at next moment. The skilled drivers will have stronger judgment on anticipation of optimal velocity and quicker response to the optimal velocity changes with memory than those unskilled drivers. However, the investigation of driver's memory on the continuum models is rare. To overcome this limitation and inspired by the memory effect, an improved continuum model will be presented to investigate optimal velocity changes with memory on traffic flow.

In this paper, an extended continuum traffic flow model with the consideration of driver's optimal velocity changes with memory is presented. We obtained the stability condition of the proposed model by using linear stability theory. The KdV–Burgers equation is derived to describe the traffic flow behavior. And the soliton solution is given. Theoretic analysis and numerical simulation has been proposed to explore this complex phenomenon resulted. Numerical simulations demonstrates that the effect of optimal velocity changes with memory can avoid the disadvantage of historical information, which increases the stability of traffic flow on road, and so it improve the traffic flow stability and minimize cars' energy consumptions.

2. The extended continuum model

In real driving situation, the driver's memory plays an important role. Considering the effect of optimal velocity changes with memory, Peng proposed a new car-following model accounting for the optimal velocity changes with memory as follows [46]:

$$\frac{dv_n(t)}{dt} = a\left[V(\Delta x_n(t)) - v_n(t)\right] + \lambda \Delta v_n(t) + \gamma\left[V(\Delta x_n(t)) - V(\Delta x_n(t-\delta))\right] \tag{1}$$

where $x_n(t)$ is the position of car n at time t, $\Delta v_n = v_{n+1} - v_n$ represents the velocity difference, $\Delta x_n = x_{n+1} - x_n$ describes the headway, $V(\Delta x_n(t))$ represents the optimal velocity function, $V(\Delta x_n(t)) - V(\Delta x_n(t-\delta))$ means the optimal velocity changes with memory, a and λ and γ are sensitivity parameters, δ indicates the memory time step. When $\gamma = 0$ or $\delta = 0$, Eq. (1) will be reduced to FVDM.

The optimal velocity $V(\Delta x_n(t))$ in Eq. (1) is chosen as follows [37]:

$$V(\Delta x_n(t)) = v_{\max}\left[\tanh(\Delta x_n(t) - h_c) + \tanh(h_c)\right]/2 \quad . \tag{2}$$

where h_c is the safety distance and $v_{\max}$ is the maximal velocity.

Before deducing the continuum model, the first-order Taylor expansion of $\Delta x_n(t-\delta)$ with neglecting nonlinear terms is deduced as follows:

$$\Delta x_n(t-\delta) = \Delta x_n(t) + \frac{d\Delta x_n(t)}{dt}(-\delta) = \Delta x_n(t) - \delta \Delta v_n(t) \tag{3}$$

For simplicity, linearization of $V(\Delta x_n(t-\delta))$ results in the following expression

$$V(\Delta x_n(t-\delta)) = V(\Delta x_n(t) - \delta\Delta v_n(t)) = V(\Delta x_n(t)) - V'(\Delta x_n(t))\,\delta\Delta v_n(t) \tag{4}$$

Putting Eq. (4) into Eq. (1), we can deduce

$$\frac{dv_n(t)}{dt} = a\left[V(\Delta x_n(t)) - v_n(t)\right] + \left(\lambda + \gamma\delta V'(\Delta x_n(t))\right)\Delta v_n(t) \tag{5}$$

Technique of converting microscopic variables into macroscopic variables is adopted as follows:

$$\begin{array}{ll} v_n(t) \to v(x,t), & v_{n+1}(t) \to v(x+\Delta, t) \\ V(\Delta x_n(t)) \to V_e(\rho), & V'(\Delta x_n(t)) \to \overline{V'}(h) \end{array} \tag{6}$$

where Δ means the distance between two adjacent vehicles, $\rho(x,t)$ and $v(x,t)$represents the macro density and macro velocity, respectively. $h = \frac{1}{\rho}$ is the mean headway, $V_e(\rho)$ indicates the equilibrium speed, $\overline{V'}(h) = -\rho^2 V_e'(\rho)$.

Make the Taylor series expansion of $v(x+\Delta, t)$ by neglecting the high-order terms, we will deduce

$$\Delta v_n(t) = v(x+\Delta, t) - v(x,t) = V'(x,t)\,\Delta + \frac{1}{2}v''(x,t)\,\Delta^2. \tag{7}$$

Putting the above macro variables into Eq. (4), the following equation is derived

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = a\left[V_e(\rho) - v\right] + \left(\lambda + \gamma\delta V'(\Delta x_n(t))\right)\left[V'(x,t)\,\Delta + \frac{1}{2}v''(x,t)\,\Delta^2\right] \tag{8}$$