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# Group learning versus local learning: Which is prefer for public cooperation?

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## HIGHLIGHTS

- This paper features the intrinsic group learning in public goods games on graphs.
- Group learning rules have remarkable performance in promoting cooperation.
- Heterogeneity does not decisively lead to a high level of cooperation.
- Separating interaction and replacement is conducive to the survival of cooperators.

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## ABSTRACT

We study the evolution of cooperation in public goods games on various graphs, focusing on the effects that are brought by different kinds of strategy donors. This highlights a basic feature of a public good game, for which there exists a remarkable difference between the interactive players and the players who are imitated. A player can learn from all the groups where the player is a member or from the typically local nearest neighbors, and the results show that the group learning rules have better performance in promoting cooperation on many networks than the local learning rules. The heterogeneity of networks' degree may be an effective mechanism for harvesting the cooperation expectation in many cases, however, we find that heterogeneity does not definitely mean the high frequency of cooperators in a population under group learning rules. It was shown that cooperators always hardly evolve whenever the interaction and the replacement do not coincide for evolutionary pairwise dilemmas on graphs, while for PG games we find that breaking the symmetry is conducive to the survival of cooperators.

## 1. Introduction

Based on the prediction of the traditional Nash equilibrium theory and evolutionary game theory, defection will be popular in prisoner's dilemma (PD) games and cooperators will die out due to defectors' free riding the common in public goods (PG) games. However, collective behaviors can be observed in a wide spread range from nature to animal or human societies [1]. The evolution of cooperation attracted many attentions of evolutionary biologists, social scientists and physicists to explore the hidden mechanisms for the generation, emergence and maintenance in many groups undergoing competitive games with various environmental conditions. In fact, over the past decades, by the efforts of many scientists, rules and mechanisms for enhancing the cooperation were found, such as the extent of kinship, direct or indirect reciprocity,

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group selection and spatial reciprocity [2–5]. Coevolution rules in pairwise interaction games, like increasing the degree or augmenting teaching activity of a player after it has successfully spread its strategy, have been shown to be effective in favor of cooperation [6–8]. These different teaching abilities and degrees of players on a graph reveals beneficial influences of heterogeneities or non-mean-field on the evolution of cooperation.

Public goods games capture the essence of conflict between the collective and personal rationalities in fulfilling a common task [9]. In a PG game, cooperators donates a unit to the public pool, while defectors contribute nothing. After the public contribution is multiplied by a synergetic enhancing ratio, the resulting amount is divided equally to all the group members irrespective their strategies. Naturally, for making a synchronous and independent decision in a well-mixed population, a rational and selfish player expect to reap the benefit generated by the investment of cooperators, this leads to "the tragedy of the commons", and no wonder Hardin prophesies many human enterprises will inescapably collapse [10]. Reward and punishment are often explored to solve the social dilemmas [11–14], while these need to confirm accurately the ones who contribute in a group. The results in [15] shows that the provisioning of public goods can be secured when even players lack strategically relevant information—individual strategy are adjusted only by reinforcement of their own past payoffs. Matthew effect, conditional strategies and tolerant strategies are also introduced to avoid the dilemmas and illustrate the complexity and diversity in public goods investing circumstance [16–18]. The optional contributions of players, the diversity of players, and the ability of inferring reputation affect the evolution of cooperation as well [19–21]. Heterogeneous or delayed distribution of payoffs are identified ways for cooperator clusters to resist the invasion of defectors, and it is shown that moderate heterogeneous distribution of payoffs promote cooperation better than strongly heterogeneous states [22,23].

The above studies focus on the mechanisms or factors in affection of cooperation without the consideration of group size and the variation of neighborhood of a player. For PD games on a regular lattice, it is shown that the interaction neighborhood with moderate size is beneficial for forming larger cooperative clusters [24]. Notably, the results in [25] shows that group size plays a key role in the evolution of PG games on a square lattice, wherein large groups can sustain cooperation by lower interest factors than small groups. By separating the interaction and replacement graphs, PD games, snowdrift games and stag-hunt games are studied, it reveals that the extent of overlap of the interaction graph and the replacement graph has a decisive effect on the survival of cooperators [26,27].

For PG games on networks, a remarkable feature is that a player will participate in many groups, then why does he/she not take advantage of any possible information in groups? Based on this point, practically, players in PG games on structured topology, have the possibility to learn each member in all groups which they once attended. On a square lattice, considering of the effects of different sizes of groups, critical enhancing factor (which cooperators die out, normalized by the group size) when strategy donors are selected from the four nearest neighbors is lower than that when strategy donors are among all the groups where the player is a member [28].

Noting the interesting characteristic of PG games and the current research states, this motivates us to investigate to what extent learning from all groups of a player where it is a member do in fact promote cooperation comparing with learning from its neighbors. On different graphs, does the graph with high heterogeneity can benefit the PG game groups using the learning rules based on the critical feature? This paper will focus on these problems. We examine the performance of two kinds of strategy updating rules, learning the selected one who has the most payoff or learning this one with an adoption probability, additionally, the selected one is distinguished between among the groups wherein the player is a member and within the group it is the focal. These rules are simulated on square lattices, nearest-neighbor coupled networks and WS small-word networks. We find that in most cases cooperation levels of learning one's all groups are significantly more than that of imitating one local group, and in the first case cooperators have the potential advantage to break through the net-shaped barrier formed by defectors. Interesting, more heterogeneity of network structures does not invariably result in larger magnitude of cooperators. Different from the fact that cooperators can hardly evolve in pairwise dilemmas if the interaction and the replacement graph are not identical, notably, separating the interaction and the replacement of players contributes to the subsistence of cooperators in PG games.

This paper is organized as follows. First, we introduce learning rules and evolutionary models in Section 2. Then, the simulation results are obtained in Section 3. Finally, we conclude the paper with a discussion in Section 4.

## 2. The model

There is a population comprising players of PG games, each individual (player) occupies a node on an undirected network with N nodes. The neighbors of a focal individual consist of the players such that each of them has a link with the focal. Each focal individual and its neighbors constitute a group (say, a PG group) to play a PG game. Clearly, the number of PG groups a player will participate in depends on the degree of this player. Since all individuals are closely associated with their neighbors as well as their neighbors' neighbors in PG games, therefore, in the part of strategy learning, we have practical reasons to consider the influence of "neighbors' neighbors" for the evolution of cooperation on different networks, and this is the paper's focus.

Take a square lattice for instance, every individual has four neighbors leads to the fact that this has four PG groups within it is a member and one group where it is the focal, and it follows that the player will play with other 12 individuals (see Fig. 1).