Accepted Manuscript

Wealth distribution, Pareto law, and stretched exponential decay of money: Computer simulations analysis of agent-based models

Ekrem Aydiner, Andrey G. Cherstvy, Ralf Metzler

PII: S0378-4371(17)30741-0
DOI: http://dx.doi.org/10.1016/j.physa.2017.08.017
Reference: PHYSA 18463

To appear in: *Physica A*

Received date : 1 April 2017
Revised date : 29 June 2017

Please cite this article as: E. Aydiner, A.G. Cherstvy, R. Metzler, Wealth distribution, Pareto law, and stretched exponential decay of money: Computer simulations analysis of agent-based models, *Physica A* (2017), http://dx.doi.org/10.1016/j.physa.2017.08.017

Wealth distribution, Pareto law, and stretched exponential decay of money: computer simulations analysis of agent-based models

Ekrem Aydiner,[1,*] Andrey G. Cherstvy,[2] and Ralf Metzler[2]

[1]*Department of Physics, Istanbul University, 34134 Istanbul, Turkey*
[2]*Institute for Physics & Astronomy, University of Potsdam, 14476 Potsdam-Golm, Germany*

(Dated: August 14, 2017)

We study by Monte Carlo simulations a kinetic exchange trading model for both fixed and distributed saving propensities of the agents and rationalize the person and wealth distributions. We show that the newly introduced wealth distribution—that may be more amenable in certain situations—features a different power-law exponent, particularly for distributed saving propensities of the agents. For open agent-based systems, we analyze the person and wealth distributions and find that the presence of trap agents alters their amplitude, leaving however the scaling exponents nearly unaffected. For an open system, we show that the total wealth—for different trap agent densities and saving propensities of the agents—decreases in time according to the classical Kohlrausch-Williams-Watts stretched exponential law. Interestingly, this decay does not depend on the trap agent density, but rather on saving propensities. The system relaxation for fixed and distributed saving schemes are found to be different.

PACS numbers: 89.65.Gh

Keywords: econophysics, wealth and income distribution, Pareto law, scaling exponents

I. INTRODUCTION

Economical inequality and long-tailed income distributions—inevitable and vital paradigms in modern societies[1–6]—offer interesting subjects of investigation for societal and physical-mathematical sciences, see also Refs.[7–9]. For a long time economists have been trying to pinpoint the exact reasons of monetary inequalities, and to quantitatively answer the question how income and wealth are distributed.

Historically, a number of attempts were made to answer these questions. More than a century ago, Pareto performed extensive studies in Europe and found that the income distributions possess power law tails[10], in contrast to the Gaussian financial market models of Regnault and Bachelier[11,12]. Mandelbrot then picked up on Pareto's ideas and coined the term *strong Pareto law* for the income distribution[13]. Gibrat worked on similar aspects of the wealth distribution and proposed a *law of proportionate growth*[14]. Gibrat realized that the Pareto's law did not fit the whole range of the data, being valid only for the "rich end". Champernowne considered the same problem proposing a probabilistic theory reproducing the Pareto's law[15].

Today, we realize that the distributions of income indeed reveal some globally stable and robust features[16–18]. In general, the bulk of the distributions of income can be fitted reasonably well by the log-normal (Boltzmann-Gibbs) and Gamma distributions[4,19–23,25]. Economists prefer the log-normal form[19,20] for the positively-defined measure of money, whereas physicists use the Gamma distribution[4,21,22]. The latter generally providing a better fit of the data[23], see also Ref.[24] There is more consensus on the upper end of the distribution described by the Pareto law, see also Refs.[25–28]. Recently, the income data analyses were extensively reviewed[3,5].

The lack of easily-available data for the wealth distribution, analogous to yearly income-tax returns offering a measure of income[26], compels researchers to resort to indirect methods. For the income distribution, the analysis indicates that[3–5,19–23,26,29,30]

$$P(m) \simeq m^{-(1+\nu)} e^{-m/m_c}, \qquad (1)$$

for incomes m smaller than a characteristic income dented m_c, and the power-law form

$$P(m) \simeq m^{-(1+\nu)}, \qquad (2)$$

for $m \geq m_c$. Here, μ and ν are the respective scaling exponents. The exponent ν is the Pareto exponent, typically ranging between one and three[22,29,31–33], so that a typical income $\langle m \rangle$ exists. For a historical overview of Pareto's data and the recent progress we refer to Refs.[5,34,35] and to the study[31] for a double power law of $P(m)$. The subject of income distribution got massive attention in the last decade[3,5,7–9,18,23,26,28–31,35–66]. The crossover point m_c in Eq. (1) can be practically extracted via fitting the initial Gamma distribution and the power-law tail in Eqs. (1) and (2)[22]. An alternative way to joint together the bulk Gamma distribution and the Pareto law for the tails is to choose m_c such that the jointed distribution is continuous and differentiable at this point.

The allocation of wealth and income in a society is the subject of intense research in econophysics. Despite the decades of development and data analysis, the precise mathematical mechanisms are not fully understood. To reveal them, a number of models were proposed based on statistical mechanics analysis, see also Ref.[67]. In particular, to explain the functioning principles of trading systems, a number of ideal-gas saving-based models of closed