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On the spectrum of plasma modes in a field-free pair plasma: Dispersion and Landau damping in Tsallis statistics

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HIGHLIGHTS

- Three types of plasma modes are confirmed, i.e., IPWs and IAWs with Landau damping, and transverse electromagnetic waves without Landau damping.
- IAWs are heavily damped.
- The supra-thermal background makes the Landau damping more effective and prominent.
- The electromagnetic modes are comparatively less sensitive to the background distribution.

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ABSTRACT

The full spectrum of possible plasma modes and their Landau damping in a field-free pair plasma are examined analytically in the context of Tsallis statistics. This study is based on the solving the linearized Vlasov equation and Maxwell's equations by employing the general method of characteristics, i.e., integrating along unperturbed orbits. Three types of plasma modes are confirmed: two of them are electrostatic waves with Landau damping, i.e., ion plasma waves (IPWs) and ion-acoustic waves (IAWs); and one mode is the transverse electromagnetic waves (light waves) without Landau damping. Our analysis shows that the Landau damping time for IAWs is negligible for most of wave lengths, and so these modes are heavily damped. Furthermore, Landau damping time for IPWs are considerable and these modes are of a great importance. Comparison of Landau damping in the case of a supra-thermal background distribution (confirmed by considering $q < 1$ for spectral index) with the ones for a Maxwellian plasma in thermal equilibrium (confirmed by considering the asymptotic limit $q \to 1$) shows that the supra-thermal background makes the Landau damping more effective and prominent and also shifts the normal modes to the regions with less wave numbers. Basically, the damping rate in a supra-thermal plasma is stronger than a Maxwellian plasma. Moreover, the electromagnetic modes are comparatively less sensitive to the background distribution.

1. Introduction

Recently, many authors have focused to the field of linear and non-linear aspects of wave phenomena in non-Maxwellian plasmas [1–7]. The most motivating reason for such an interest is maybe a number of astrophysical and experimental observations which indicate that the velocity distribution function of particles in plasmas do not obey exactly the standard Maxwell–Boltzmann distribution, but it shows deviations from thermal distribution (see, e.g., the Refs. [8–13]). So, based

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on these observations some people have proposed the models that would describe the non-thermal behavior of particles in space plasma environments and laboratory experiments, like the so-called Kappa (κ) distribution, which was introduced for the first time by Vasyliunas in 1968 [14] for describing plasmas far from the thermal equilibrium such as the Magnetosphere environment and the Solar winds (see, e.g., [15]); or the non-thermal model advanced by Cairns et al. in 1995 [16], which was introduced to explain the solitary electrostatic structures involving density depletions that have been observed in the upper ionosphere in the auroral zone by the Freja satellite [17]; and the q-nonextensive model, which was proposed by Tsallis in 1988 [18] as a suitable generalization of the Boltzmann–Gibbs–Shannon (BGS) statistics. The common feature of the velocity distribution function in these models is an extended spectrum for particles speed which could display the super-thermal or sub-thermal regions for the particles that have deviated from thermal distribution. Such velocity distribution functions show non-Maxwellian tails decreasing as a power-law distribution in particle speed. For our purpose, we choose the Tsallis non-extensive model, with a powerful thermostatistics foundation [19–21] and considerable experimental evidences [22–27], that cover many features of the other non-thermal models. We can generally say that systems subject to long-range interactions and correlations, systems with long-time memories, and fractal space–time structures are related to the non-Maxwellian statistics where the standard Boltzmann–Gibbs statistics collapses [28–31]. The Tsallis entropy as a non-extensive generalization of the BGS entropy is given by expression $S_q = k_B \frac{1-\sum_i p_i^q}{q-1}$ [18], where k_B is the standard Boltzmann constant, $\{p_i\}$ denotes the probabilities of the microstate configurations and q is a real parameter quantifying the degree of the nonextensivity. The most distinctive feature of S_q is its pseudoadditivity. Given a composite system $A + B$, constituted by two subsystems A and B, which are independent in the sense of factorizability of the joint microstate probabilities, the Tsallis entropy of the composite system $A+B$ satisfies $S_q(A+B) = S_q(A)+S_q(B)+(1-q)S_q(A)S_q(B)$. In the limit of $q \to 1$, S_q reduces to the celebrated logarithmic BGS entropy $S = -k_B\sum_i p_i \ln p_i$, and the usual additivity of the entropy is recovered. Hence, $|1 - q|$ is a measure of the lack of extensivity of the system.

Among a variety of problems in plasma physics, there is a considerable attention to the elementary properties of pair plasmas, because of its significance in astrophysics and laboratory experiments (see, e.g., the Refs. [32–36]). Pair plasmas is classified as a equal mass multi-species plasma, which its physical properties are different from those of the ordinary electron–ion plasmas. There is no parity between time-space scales of the positive- and negative-charged particles in a pair plasma, because the mobility of the particles in the electromagnetic fields are the same. Pair plasmas consisting of electrons and positrons exist in astrophysical situations like active galactic nuclei (AGN) [37], pulsar and neutron star magnetosphere [38], solar atmosphere [39], accretion disk [40], black holes [41], the early universe [42]. Moreover, generation of the electron–positron plasma is possible in laboratory experiments by β^+ unstable isotopes and electron beams in plasma traps [43–45], or by interaction of ultra intense laser pulses with solid targets [46]. Furthermore, the generation of pure pair-ion plasmas consisting of only positive and negative ions with equal masses, i.e., the fullerenes C_{60}^- and C_{60}^+ [47], opened a new field for diagnostic of waves and instabilities in pure pair-ions plasmas [48].

The objective of the present paper is to present a fully kinetic analysis for study of the complete range of possible plasma waves in a field-free pair plasma with emphasizing the Tsallis statistics. The motivation of this work arises from some theoretical papers that had predicted the only wave modes in the case of an unmagnetized pure pair plasma are the electrostatic Langmuir waves and the electromagnetic transverse modes (see, e.g., [32,49,50]). This is in contrast with the experimental examination of the electrostatic waves in a pure pair-ion plasma, where properties of the wave propagation along the B-field lines have been measured by Oohra et al. [48]. In the mentioned work, three electrostatic modes in the measured dispersion relation have been reported; $\frac{\omega}{2\pi} < 8\ kHz$, $8\ kHz < \frac{\omega}{2\pi} < 32\ kHz$, and $\frac{\omega}{2\pi} > 32\ kHz$, where they refer to the modes as the ion acoustic waves (IAWs), the intermediate-frequency waves (IFWs), and the ion plasma waves (IPWs), respectively. There, electromagnetic modes relevant to the plasma were be neglected because the density and the temperature were relatively low and the induction current of the ions was very small. It is to be mentioned that IFWs has a feature that the group velocity is negative but the phase velocity is positive, i.e., the mode is like a backward wave.

Here, we want to use the general method of characteristics, i.e., integrating along unperturbed orbits, for deriving the full range of possible plasma waves and their Landau damping in a field-free pair plasma. Recently, the problem of Langmuir oscillations in an electron–positron plasma has been studied by employing a simplified solution to the Vlasov–Poisson equations in Tsallis statistics [51]. Particularly, our aim here is derivation of the compact expressions for dispersion relations and imaginary parts of all the possible eigenfrequencies in a general pair plasma in the context of Tsallis statistics. The results must predict IAWs and transverse electromagnetic waves (light waves) besides the IPWs (Langmuir waves), and also they have to contain Landau damping of the plasma modes. Finally, our solutions have to be flexible to reduce to the ones for a Maxwellian pair plasma.

The paper is structured as follows. Section 2 deals with the model equations and general method of characteristics for deriving the dispersion tensor D in a field-free pair plasma. In Section 3, the solution of $D \cdot \bar{E} = 0$ is discussed in the context of Tsallis statistics, which leads to full range of possible plasma modes. The discussion and some comments are presented in Section 4. Finally, the paper is summarized in Section 5.

2. Model equations and dielectric tensor

We consider the linearized Vlasov equation in a field-free pair plasma as [52]:

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}_x\right) f_{\alpha 1}(\vec{x}, \vec{v}, t) = -\frac{q_\alpha}{m_\alpha}\left[\vec{E}_1 + \frac{\vec{v}\times\vec{B}_1}{c}\right]\cdot\vec{\nabla}_v f_{\alpha 0}(\vec{v}), \tag{1}$$